IRAQI STATISTICIANS JOURNAL

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Comparing some estimation methods for Frechet Distribution (Simulation)

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ARTICLE INFO

Article history:

Received	18 February 2024
Revised	19 February 2024
Accepted	13 March 2024
Available online	13 March 2024

Keywords:
Frechet distribution,
Maximum Likelihood Estimation (MLE),
Method of Moments (MoM),
Least Squares Estimation (LSE),
Least Absolute Difference,
mean squared error

ABSTRACT

The Frechet distribution, widely utilized in various fields such as hydrology, finance, and environmental sciences, poses challenges in parameter estimation, This study aims to compare several estimation methods for the Frechet distribution under scenarios with outlier observations. We consider both classical and robust estimation techniques, including (Maximum Likelihood Estimation (MLE), Method of Moments (MOM), Least Squares Estimation (LSE), and Robust Regression methods(RRE)). To evaluate the performance of these methods, extensive simulation studies are conducted under different sample sizes and intial parameter values .mean squared error, are employed to assess the accuracy and robustness of each method under varying conditions. Our findings provide insights into the effectiveness and limitations of different estimation approaches for the Frechet distribution when simulation parameters are present, aiding practitioners in selecting suitable methods for robust parameter estimation in practical applications.

Simulation results show that estimation method effected with (sample size, initial parameter values and evaluation criteria) , Bayesian estimation methods can be compared with Non-Bayesian estimation methods for Frechet distribution.

1. Introduction

The Frechet distribution is the basic model of extreme value theory and plays a central role in various fields such as hydrology, finance and environmental science, and engineering. Its ability to model extreme events such as floods, stock market crashes, and environmental disasters makes it a crucial tool for risk assessment and decision-making. However, the accurate estimation of its parameters remains a challenging task for this reason many research's was presented such that Parameter estimation plays a crucial role in determining the characteristics of a probability distribution [1]. As a result, numerous estimation methods have been extensively studied in the field of statistics. In this article, six different methods were examined to estimate the parameters of the Frechet distribution.

This research aims to address this challenge by comparing several estimation methods for the Frechet distribution in the presence of outlier cases. We consider both classical and robust estimation approaches, evaluating their performance under various outlier scenarios and sample sizes. The comparative analysis includes well-established methods such as MLE and MOM, as well as robust techniques such as Least Squares Estimation (LSE) and Robust Regression methods (RRE).

we seek to assess the accuracy, efficiency, and robustness of each estimation method. mean squared error will be employed to evaluate the performance of the estimation techniques under consideration.

The findings of this study are provide valuable insights into the suitability of different estimation methods for robust parameter estimation of the Frechet distribution .

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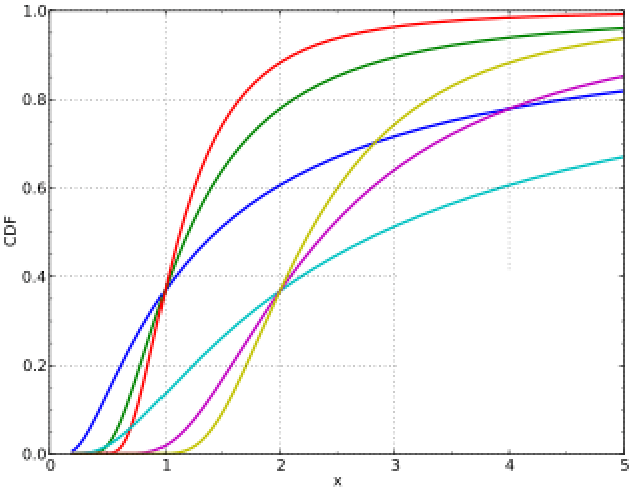
2. Fréchet distribution

The Fréchet distribution, named after Maurice Fréchet, is a type of probability distribution used in the field of extreme value theory. It is often employed to model the behavior of extreme events, such as the maximum value reached by a phenomenon over a given period. The Fréchet distribution is characterized by its heavy right tail, making it suitable for modeling rare but extreme events.
Cumulative distribution function (CDF) of the Fréchet distribution is given by [4]

$$F(x;\alpha,\beta,\gamma)=\begin{cases} e^{-\left(\frac{x-\gamma}{\beta}\right)^{-\alpha}} & x>\gamma \\ 0 & x\le\gamma \end{cases} \qquad (1)$$



Fig(1) the cumulative distribution function[4]

$(\alpha>0)$ is the shape parameter, determining the heaviness of the tail.
$(\beta>0)$ is the scale parameter, controlling the scale of the distribution.
(γ) is the location parameter, representing the minimum value of the distribution.
Probability density function (PDF) of the Fréchet distribution is given by :-[2]

$$f(x;\alpha,\beta,\gamma)=\left(\frac{\alpha}{\beta}\right)\left(\frac{x-\gamma}{\beta}\right)^{-(1+\alpha)} e^{-\left(\frac{x-\gamma}{\beta}\right)^{-\alpha}} \qquad (2)$$

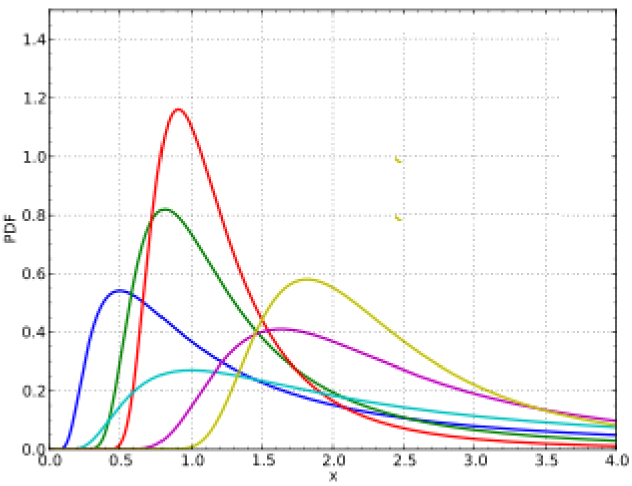


Fig.(2) the probability density function [4]

3. Estimation Methods

a. Maximum Likelihood Estimation (MLE)

Maximum Likelihood Estimation (MLE) is a widely used method for estimating the parameters of a probability distribution by maximizing the likelihood function, which measures the likelihood of observing the given data under a particular set of parameter values.

For the Fréchet distribution, let's assume we have a sample $(x_1 \;, x_2 \,, \ldots, x_n \;)$ of independent and identically distributed (i.i.d.) random variables following a Fréchet distribution.

The likelihood function $(L(\alpha, \beta, \gamma))$ for the Fréchet distribution is the product of the PDF evaluated at each observation:[8]

$$L(\alpha, \beta, \gamma) = \prod_{i=1}^{n} f(x; \alpha, \beta, \gamma) \qquad \ldots (3)$$

Taking the natural logarithm of the likelihood function, we get the log-likelihood function

$$\ell(\alpha, \beta, \gamma) = \sum_{i=1}^{n} \left[- \log(\beta) - (1+\alpha) \log\left(\frac{x_i - \gamma}{\beta}\right) - \left(\frac{x_i - \gamma}{\beta}\right)^{-\alpha}\right] + n * \log(\alpha) \qquad \ldots (4)$$

To estimate the parameters $(\alpha, \beta \;\; and\, \gamma \;)$ we maximize the log-likelihood function numerically, typically using optimization algorithms such as the Newton-Raphson method.

b. Method of Moment (MOM)

The Method of Moments (MoM) is another common technique for estimating the parameters of a probability distribution. In the context of the Fréchet distribution, we can use MoM to estimate the parameters (α, β, γ)

For the Fréchet distribution, the moments are defined as follows:[5]

The first raw moment (mean) is given by:

$$\mu_1 = \gamma + \frac{\beta}{1 - \frac{1}{\alpha}} \qquad \ldots (5)$$

The second central moment (variance) is given by:

$$\mu_2 = \frac{\beta^2}{(1 - \frac{2}{\alpha})(1 - \frac{1}{\alpha})^2} \qquad \ldots (6)$$

Using the sample moments (sample mean and sample variance), we can solve for the parameters (α, β, γ)

Calculate the sample mean $(\bar{x})$ and sample variance $(s^2 \;)$ from the observed data then estimate the location parameter (γ) as $(\bar{x})$ then Estimate the scale parameter $(\beta \;)$ using the relationship between the sample variance and the theoretical variance:

$$s^2 = \frac{\beta^2}{(1 - \frac{2}{\alpha})(1 - \frac{1}{\alpha})^2} \qquad \ldots (7)$$

Ones $(\gamma \; and \; \beta \;)$ are estimated, solve for (α) using the first moment equation:

$$\alpha = \frac{1}{1 - \frac{\beta}{\bar{x} - \gamma}} \qquad \ldots (8)$$

These estimates provide the Method of Moments estimators for the Fréchet distribution parameters.

c. Least Squares Estimation (LSE)

Least Squares Estimation (LSE) is a technique commonly used for parameter estimation in statistical modeling. In the context of the Fréchet distribution, we can employ LSE to estimate the parameters (α, β, γ) by minimizing the sum of squared differences between the observed data and the corresponding Fréchet distribution quantiles.

The quantile function (inverse cumulative distribution function) of the Fréchet distribution is given by [6]

$$Q(p; \alpha, \beta, \gamma) = \gamma + \beta[-\log(p)]^{-\frac{1}{\alpha}} \quad \ldots (9)$$

Where $(0 < p < 1)$ is the probability

Let $(x_{(1)}, x_{(2)}, \ldots, x_{(n)})$ denote the ordered sample values

To estimate the parameters using LSE, we minimize the sum of squared differences between the observed quantiles and the corresponding theoretical quantiles

The objective function to be minimized is:

$$\text{Minimize } \left(\sum_{i=1}^{n}\left[Q\left(\frac{i}{n+1}; \alpha, \beta, \gamma\right) - x_{(i)}\right]^2\right) \quad \ldots (10)$$

We can use optimization algorithms such as gradient descent, Newton's method, or other numerical optimization techniques to minimize this objective function and obtain the estimates for (α, β, γ)

Once the optimization process converges, the resulting parameter estimates provide the least squares estimates for the parameters of the Fréchet distribution.

d. Robust Regression estimation (RRE)

Robust regression methods are particularly useful when dealing with data that may contain outliers or deviations from the assumed distributional form. In the context of estimating parameters for the Fréchet distribution, robust regression techniques can help mitigate the influence of outliers and provide more reliable parameter estimates. Here, we'll discuss M-estimation in robust regression methods [7-11].

M-estimation involves minimizing a robust objective function that is less sensitive to outliers compared to traditional least squares estimation. One commonly used objective function is the Huber loss function, which combines a quadratic loss for small residuals and a linear loss for large residuals:

$$\sum_{i=1}^{n} \rho\left(\frac{y_i - f(x_i; \theta)}{\sigma}\right) \quad \ldots (11)$$

Where $(\rho(z))$ is a robust loss function, (y_i) are the observed data

$f(x_i; \theta)$ is the Fréchet distribution quantile function with parameters (θ) and (σ) is a scale parameter.

4. Comparison metrics

The measure was assumed to be the least absolute deviation (LAD), according to the following formula [9]

$$\omega_j = Min\left|\hat{\vartheta}_j - \vartheta_j\right| \quad \ldots (12)$$

Such that

ω_j Represent the (LAD), ϑ_j Represent the true value and $\hat{\vartheta}_j$ Represent the estimated value

$$\text{LAD} = Min(\omega_j) \quad \ldots (13)$$

The other measure was assumed to be the mean square error (MSE), according to the following formula:-

$$MSE = \frac{\sum_{it=1}^{rep}[\hat{\vartheta}_j - \vartheta_j]^2}{rep} \quad \ldots (14)$$

rep Represent Number of iterations

5. Simulation parameters

The initial parameter values for the simulation experiments was

The sample size will be $(n_1 = 50, n_2 = 100, n_3 = 150)$, Shape parameter $(\alpha_1 = 0.25, \alpha_2 = 0.5)$, Scale parameter $(\beta_1 = 0.5, \beta_2 = 1, \beta_3 = 1.5)$ and Location parameter $(\gamma_1 = 2, \gamma_2 = 3)$

The generation of Frechet random samples by the following form:-

$$x = \gamma + \beta \left[\frac{-1}{Ln(R)}\right]^{\frac{1}{\alpha}} \quad \ldots (15)$$

Such that (R) is a random generation.

6. Experimental Results

After simulation processes with initial parameters for all experiments the following results was shown [0,1].

Table (1) shows the estimated values for all methods and for various experiments showed estimated values close to the true values

Table (1) the estimated values for all parameters of Fréchet distribution

α	β	γ	n	$\hat{\alpha}_{MLE}$	$\hat{\beta}_{MLE}$	$\hat{\gamma}_{MLE}$	$\hat{\alpha}_{MOM}$	$\hat{\beta}_{MOM}$	$\hat{\gamma}_{MOM}$	$\hat{\alpha}_{LSE}$	$\hat{\beta}_{LSE}$	$\hat{\gamma}_{LSE}$	$\hat{\alpha}_{RRE}$	$\hat{\beta}_{RRE}$	$\hat{\gamma}_{RRE}$
0.25	0.5	2	50	0.2537	0.4581	1.9903	0.3221	0.5100	2.0194	0.2812	0.4891	2.0191	0.2917	0.4403	1.9965
0.25	0.5	2	100	0.2515	0.6275	2.0031	0.2490	0.5007	2.0062	0.2454	0.6333	2.0014	0.2480	0.6345	2.0055
0.25	0.5	2	150	0.2430	0.4986	2.0069	0.2506	0.5000	2.0050	0.2430	0.4980	2.0036	0.2434	0.4982	2.0039
0.25	0.5	3	50	0.2023	0.3990	2.9713	0.2387	0.5248	3.0447	0.2030	0.4005	2.9856	0.2553	0.3775	2.9857
0.25	0.5	3	100	0.3391	0.6793	3.0049	0.2520	0.4975	3.0065	0.3368	0.6738	3.0078	0.3435	0.6793	3.0035
0.25	0.5	3	150	0.2501	0.4714	3.0022	0.2502	0.5028	3.0056	0.2498	0.4722	3.0036	0.2506	0.4717	3.0041
0.25	1	2	50	0.3278	1.3658	2.0202	0.2538	1.0598	2.0482	0.3080	1.3650	1.9613	0.2621	1.4112	1.9636
0.25	1	2	100	0.3643	0.9814	2.0034	0.2493	0.9999	2.0004	0.3533	0.9809	2.0011	0.3566	0.9832	2.0061
0.25	1	2	150	0.2879	1.0938	2.0035	0.2501	1.0001	2.0051	0.2884	1.0938	2.0024	0.2880	1.0944	2.0024
0.25	1	3	50	0.2344	0.7651	3.0153	0.2451	0.9643	2.9855	0.2177	0.7664	2.9868	0.2889	0.7919	3.0286
0.25	1	3	100	0.3941	0.9593	3.0043	0.2501	1.0033	3.0046	0.3909	0.9548	3.0007	0.3913	0.9577	3.0113
0.25	1	3	150	0.2706	0.8634	3.0046	0.2503	0.9996	3.0024	0.2710	0.8635	3.0079	0.2708	0.8643	3.0022
0.25	1.5	2	50	0.2508	1.6929	1.9785	0.2260	1.4837	2.0291	0.2426	1.6976	2.0285	0.2827	1.7767	2.0011
0.25	1.5	2	100	0.2436	1.1140	2.0063	0.2495	1.5022	2.0030	0.2412	1.1164	1.9990	0.2460	1.1142	1.9958
0.25	1.5	2	150	0.2421	1.5903	2.0034	0.2498	1.5002	2.0078	0.2416	1.5907	2.0064	0.2421	1.5902	2.0056
0.25	1.5	3	50	0.2338	1.6209	3.0414	0.2560	1.5291	3.0143	0.2548	1.5440	3.0503	0.2115	1.5592	3.0320
0.25	1.5	3	100	0.2340	1.8181	3.0060	0.2470	1.5030	2.9987	0.2348	1.8146	3.0031	0.2416	1.8191	2.9992
0.25	1.5	3	150	0.2711	1.5210	3.0048	0.2499	1.4999	3.0060	0.2714	1.5202	3.0042	0.2706	1.5206	3.0017
0.5	0.5	2	50	1.0735	0.6345	2.0403	0.5513	0.5138	2.0199	1.0934	0.5991	1.9978	1.0227	0.5577	1.9827
0.5	0.5	2	100	0.8051	0.6012	2.0080	0.5008	0.4967	2.0073	0.8162	0.5996	2.0013	0.8111	0.5983	2.0062
0.5	0.5	2	150	1.2583	0.5116	2.0040	0.4999	0.5002	2.0043	1.2583	0.5127	2.0033	1.2583	0.5125	2.0057
0.5	0.5	3	50	0.4934	0.4598	2.9829	0.4576	0.5000	3.0102	0.4799	0.4285	2.9657	0.5402	0.4042	2.9902
0.5	0.5	3	100	0.4935	0.4450	3.0046	0.4957	0.5018	3.0078	0.4948	0.4378	3.0097	0.5017	0.4402	3.0049
0.5	0.5	3	150	0.5001	0.4713	3.0012	0.4995	0.5000	3.0059	0.4998	0.4710	3.0009	0.5002	0.4717	3.0052
0.5	1	2	50	0.7741	1.1202	2.0802	0.4931	0.9635	2.0365	0.8606	1.1097	1.9900	0.8728	1.0546	2.0076
0.5	1	2	100	0.5388	1.2605	2.0047	0.4997	0.9941	2.0043	0.5371	1.2537	2.0061	0.5389	1.2622	2.0048
0.5	1	2	150	0.6519	0.9851	2.0030	0.5002	0.9996	2.0066	0.6516	0.9856	2.0024	0.6518	0.9859	2.0063
0.5	1	3	50	0.5552	0.9006	3.0139	0.4898	0.9868	2.9698	0.5151	0.9872	3.0205	0.5426	0.9568	2.9966
0.5	1	3	100	0.5910	1.0312	3.0008	0.4990	0.9952	3.0009	0.5924	1.0376	3.0031	0.5998	1.0392	3.0031
0.5	1	3	150	0.4970	1.0091	3.0049	0.4997	1.0005	3.0057	0.4975	1.0091	3.0057	0.4973	1.0094	3.0051
0.5	1.5	2	50	0.4669	1.1396	2.0464	0.4896	1.4599	1.9988	0.4916	1.1055	2.0278	0.4898	1.0832	1.9748
0.5	1.5	2	100	0.5636	1.7847	2.0113	0.5008	1.4964	1.9998	0.5712	1.7843	2.0019	0.5739	1.7784	2.0048
0.5	1.5	2	150	0.5206	1.3650	2.0046	0.4998	1.5006	2.0036	0.5207	1.3648	2.0057	0.5202	1.3655	2.0044
0.5	1.5	3	50	0.5605	1.5366	3.0310	0.4727	1.5211	3.0240	0.5711	1.5429	2.9851	0.5929	1.5200	3.0108
0.5	1.5	3	100	0.4755	1.7655	3.0069	0.5065	1.4998	3.0047	0.4759	1.7629	3.0023	0.4762	1.7606	3.0028
0.5	1.5	3	150	0.6417	1.4812	3.0071	0.5000	1.5002	3.0043	0.6425	1.4808	3.0023	0.6419	1.4814	3.0042

Table (2) shows the LAD values for the various experiments showed estimated values close to the true values, and that the best method, based on the total number of experiments, is the MLE, with a percentage of(58%).

Table (2) the least absolute deviation (LAD) for all parameters of Fréchet distribution

				ω_α		ω_β		ω_γ		LAD	
α	β	γ	n	Value	No	Value	No	Value	No	Value	No
0.25	0.5	2	50	3.70E-03	1	1.00E-02	2	3.45E-03	4	3.45E-03	3
0.25	0.5	2	100	1.02E-03	2	6.67E-04	2	1.44E-03	3	6.67E-04	2
0.25	0.5	2	150	5.60E-04	2	3.72E-06	2	3.63E-03	3	3.72E-06	2
0.25	0.5	3	50	5.28E-03	4	2.48E-02	2	1.43E-02	4	5.28E-03	1
0.25	0.5	3	100	2.04E-03	2	2.51E-03	2	3.47E-03	4	2.04E-03	1
0.25	0.5	3	150	7.54E-05	1	2.83E-03	2	2.15E-03	1	7.54E-05	1
0.25	1	2	50	3.77E-03	2	5.98E-02	2	2.02E-02	1	3.77E-03	1
0.25	1	2	100	7.48E-04	2	7.24E-05	2	4.17E-04	2	7.24E-05	2
0.25	1	2	150	9.33E-05	2	1.16E-04	2	2.37E-03	3	9.33E-05	1
0.25	1	3	50	4.92E-03	2	3.57E-02	2	1.32E-02	3	4.92E-03	1
0.25	1	3	100	7.81E-05	2	3.29E-03	2	7.39E-04	3	7.81E-05	1
0.25	1	3	150	3.28E-04	2	4.11E-04	2	2.21E-03	4	3.28E-04	1

0.25	1.5	2	50	7.64E-04	1	1.63E-02	2	1.12E-03	4	7.64E-04	1
0.25	1.5	2	100	5.15E-04	2	2.24E-03	2	1.00E-03	3	5.15E-04	1
0.25	1.5	2	150	2.09E-04	2	2.35E-04	2	3.38E-03	1	2.09E-04	1
0.25	1.5	3	50	4.83E-03	3	2.91E-02	2	1.43E-02	2	4.83E-03	1
0.25	1.5	3	100	2.95E-03	2	2.96E-03	2	7.68E-04	4	7.68E-04	3
0.25	1.5	3	150	1.11E-04	2	9.79E-05	2	1.74E-03	4	9.79E-05	2
0.5	0.5	2	50	5.13E-02	2	1.38E-02	2	2.17E-03	3	2.17E-03	3
0.5	0.5	2	100	7.57E-04	2	3.28E-03	2	1.25E-03	3	7.57E-04	1
0.5	0.5	2	150	1.27E-04	2	1.61E-04	2	3.32E-03	3	1.27E-04	1
0.5	0.5	3	50	6.59E-03	1	1.98E-05	2	9.77E-03	4	1.98E-05	2
0.5	0.5	3	100	1.66E-03	4	1.77E-03	2	4.60E-03	1	1.66E-03	1
0.5	0.5	3	150	6.59E-05	1	2.24E-05	2	8.80E-04	3	2.24E-05	2
0.5	1	2	50	6.95E-03	2	3.65E-02	2	7.64E-03	4	6.95E-03	1
0.5	1	2	100	3.21E-04	2	5.89E-03	2	4.27E-03	2	3.21E-04	1
0.5	1	2	150	1.58E-04	2	4.36E-04	2	2.35E-03	3	1.58E-04	1
0.5	1	3	50	1.02E-02	2	1.28E-02	3	3.44E-03	4	3.44E-03	3
0.5	1	3	100	9.50E-04	2	4.85E-03	2	8.04E-04	1	8.04E-04	3
0.5	1	3	150	2.50E-04	2	5.19E-04	2	4.87E-03	1	2.50E-04	1
0.5	1.5	2	50	8.36E-03	3	4.01E-02	2	1.22E-03	2	1.22E-03	3
0.5	1.5	2	100	7.77E-04	2	3.61E-03	2	1.75E-04	2	1.75E-04	3
0.5	1.5	2	150	1.90E-04	2	5.57E-04	2	3.63E-03	2	1.90E-04	1
0.5	1.5	3	50	2.73E-02	2	2.00E-02	4	1.08E-02	4	1.08E-02	3
0.5	1.5	3	100	6.47E-03	2	1.75E-04	2	2.26E-03	3	1.75E-04	2
0.5	1.5	3	150	3.30E-05	2	2.08E-04	2	2.31E-03	3	3.30E-05	1

Table (3) shows the MSE values for the various experiments showed estimated values close to the true values

Table (3) the Minimum mean square error for all parameters of Fréchet distribution

				$Min-MSE_{\alpha}$		$Min-MSE_{\beta}$		$Min-MSE_{\gamma}$	
α	β	γ	n	Value	No.	Value	No.	Value	No.
0.25	0.5	2	50	1.09E-03	3	9.26E-04	2	2.93E-05	4
0.25	0.5	2	100	3.06E-06	1	1.61E-06	2	1.09E-05	1
0.25	0.5	2	150	4.32E-07	2	2.12E-08	2	2.60E-05	3
0.25	0.5	3	50	3.55E-05	4	1.75E-03	2	7.52E-04	4
0.25	0.5	3	100	4.18E-06	2	7.09E-06	2	4.90E-05	1
0.25	0.5	3	150	2.56E-08	2	3.53E-07	2	1.12E-05	1
0.25	1	2	50	1.79E-04	4	3.98E-03	2	1.32E-03	4
0.25	1	2	100	1.92E-05	2	2.30E-06	2	1.05E-05	3
0.25	1	2	150	6.27E-08	2	1.57E-08	2	5.93E-06	4
0.25	1	3	50	1.05E-03	3	3.47E-03	2	5.41E-04	3
0.25	1	3	100	2.93E-05	2	1.67E-05	2	7.80E-06	3
0.25	1	3	150	1.19E-07	2	1.76E-07	2	5.01E-06	4
0.25	1.5	2	50	1.28E-04	1	6.07E-03	2	2.64E-04	4
0.25	1.5	2	100	4.73E-07	2	5.51E-05	2	1.36E-05	3
0.25	1.5	2	150	4.78E-08	2	4.66E-07	2	1.31E-05	1
0.25	1.5	3	50	1.53E-04	2	1.44E-03	2	4.00E-04	2
0.25	1.5	3	100	1.99E-05	2	1.94E-05	2	6.01E-07	4
0.25	1.5	3	150	1.85E-07	2	2.39E-08	2	4.20E-06	4
0.5	0.5	2	50	2.63E-03	2	2.14E-03	2	2.08E-05	3
0.5	0.5	2	100	4.15E-06	2	1.20E-05	2	4.07E-06	3
0.5	0.5	2	150	7.41E-08	2	2.61E-07	2	1.10E-05	3
0.5	0.5	3	50	4.20E-04	3	8.48E-06	2	1.18E-03	3
0.5	0.5	3	100	1.22E-05	4	3.08E-05	2	5.62E-05	4
0.5	0.5	3	150	4.56E-09	1	1.28E-08	2	7.89E-07	3
0.5	1	2	50	3.37E-04	2	2.10E-03	2	1.18E-04	4
0.5	1	2	100	2.01E-06	2	3.50E-05	2	2.32E-05	1
0.5	1	2	150	3.26E-08	2	3.47E-07	2	9.04E-06	3
0.5	1	3	50	6.38E-04	2	1.98E-04	2	2.50E-05	4
0.5	1	3	100	2.46E-05	2	2.54E-05	2	2.28E-06	1
0.5	1	3	150	1.61E-07	2	3.31E-07	2	2.55E-05	1
0.5	1.5	2	50	1.65E-04	3	1.80E-03	2	1.14E-03	4
0.5	1.5	2	100	8.13E-05	2	1.30E-05	2	2.44E-06	2
0.5	1.5	2	150	5.94E-07	2	3.12E-07	2	1.38E-05	2
0.5	1.5	3	50	5.44E-03	2	5.47E-04	2	2.78E-04	3
0.5	1.5	3	100	4.24E-05	2	3.36E-05	2	3.16E-05	2
0.5	1.5	3	150	1.87E-07	2	2.76E-07	2	7.48E-06	3

The best estimation method according to (α) parameter was (MOM) with percentage (72%), and the best estimation method according to (β) parameter was (MOM) with percentage (100%),and the best estimation method according to (γ) parameter was (LSE) with percentage (36%) .

7. Conclusions and Suggestions

1. The estimation method is affected by both the sample size and the initial values of the estimator
2. all estimation methods showed estimator values close to the true values according to the estimator values and their mean square errors
3. the best method is influenced by a formula with a comparison scale
4. adopting LAD criteria shows that best estimation method was MLE
5. adopting MSE criteria shows that best estimation method was MOM for each of (α, β) parameters, and was LSE for (γ) parameter
6. Bayesian estimation methods can be compared with Non-Bayesian estimation methods for Frechet distribution
7. Developing estimation methods towards applied functions (reliability function, survival function) and others
8. Real data collection and testing for Frechet distribution for application purpose

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