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An Optimized ARDL Model for Estimating and Forecasting the Dynamic Relationship between Public Revenues and Expenditures

Sulaiman Hussien Jawad[1], Ph.D. Munaf Yousif. Hmood[2]

[1,2]Department of Statistics, College of Administration and Economics, University of Baghdad Iraq

[1]E-mail of : S.H. Jawad :suliaman.hussien83@gmail.com

[2]E-mail of : M.Y. Hmood :munaf.yousif@coadec.uobaghdad.edu.iq

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ABSTRACT

Public revenues constitute the primary source for financing government expenditures and fulfilling fiscal responsibilities, and they play a crucial role in influencing a country's economic growth. This study examines the relationship between public revenues and public expenditures by integrating the classical Autoregressive Distributed Lag (ARDL) model with two artificial intelligence–based optimization algorithms: the Grey Wolf Optimizer (GWO) and the Whale Optimization Algorithm (WOA). These nature-inspired meta-heuristic algorithms mimic the cooperative hunting strategies of grey wolves and humpback whales, respectively. The integration of these algorithms with the ARDL framework aims to enhance parameter estimation, particularly when nonlinear patterns may exist in the relationship between variables, while also relaxing some of the restrictive assumptions associated with the Ordinary Least Squares (OLS) estimation method commonly used in ARDL models. Empirical findings reveal that the optimized ARDL-GWO and ARDL-WOA models substantially outperform the conventional ARDL model in out-of-sample forecasting, achieving lower RMSE and MAE values and higher directional accuracy, while the Diebold–Mariano test confirms their superior forecasting performance with no statistically significant difference between the two optimized models.

1. Introduction

The ARDL bounds-testing technique can be regarded as one of the most significant co-integration tests. Pesaran et al. [1] Developed it in 2001 to overcome a number of limitations of the traditional co-integration tests. Some of these restrictions include the need of integrating all variables of equal order, the low performance of conventional tests in small samples, and the low ability of tests to capture short-run dynamics. The strength of this methodology is that it is more efficient with small samples $(T < 50)$, and it can be used to study both short-run and long-run relationships between variables, whether they are integrated of the same order or of mixed orders, $I(0)$ and $I(1)$, as long as none of the variables is integrated of order two ,$I(2)$ [2]. Moreover, the ARDL model also gives the flexibility in the selection the appropriate lag length of each of the variables in the model to have a better grasp of how the independent variables dynamically impact on the dependent variable. It also records a part of the impacts of omitted variables and it corrects the issue of serial correlation among the residual and the issue of endogeneity created by the correlation that exists between the regressors and the residual [3].

However, this approach assumes that the relationships between variables are linear- a fact that is hardly ever true particularly when economic relationships are involved. Moreover, the ARDL technique is based on Ordinary Least Squares (OLS) to estimate the model coefficients, and OLS belongs to gradient-based algorithms, whose goal is to reduce the summation of squared errors to the highest extent possible. Nevertheless, the solution obtained is a local optimum and not necessarily the global optimum of the feasible function space, particularly when the objective function is no smooth, complex, nonlinear, or cannot be accurately differentiated. To find the best solution -requiring

Corresponding author E-mail address: suliaman.hussien83@gmail.com

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the minimization of the MSE in the function space- one needs to use algorithms that are non-derivative or gradient-based, also referred to as the gradient-free algorithms [4]. Some of these algorithms include the artificial intelligence-based optimization algorithms, which apply computational intelligence to resolve optimization problems and achieve the optimal solution or a combination of optimal solutions, in particular, where the area of search is either unknown or extremely complicated [5].

It is based on this issue that the current study seeks to refine the estimation of the Autoregressive Distributed Lag (ARDL) model through application of computational intelligence algorithms. These current meta-heuristic methods are based on randomness whereby they produce many solutions in the feasible space of the functions and therefore guarantee the determination of the best parameter estimates to the model. The coefficients obtained from the conventional ARDL model using Ordinary Least Squares (OLS) method are not thrown away in the proposed Enhanced ARDL model. Rather, these coefficients are used as initial values in the optimization process using metaheuristic algorithms. Then, the Grey Wolf Optimizer (GWO) and Whale Optimization Algorithm (WOA) are used iteratively to optimize these parameters through minimizing the Mean Squared Error (MSE). Therefore the OLS method is used as a theoretically sound base approach and the optimization algorithms are used as a follow-up enhancement stage to enhance the forecasting accuracy and not as a replacement to the standard approach of the ARDL model.

The Subsequently, the hybrid models are compared with the conventional ARDL model based on a set of forecasting performance measures, including the Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Directional Accuracy (DA), and the Diebold–Mariano (DM) test hybrid models are then contrasted to the conventional ARDL model using the Root Mean Squared error (RMSE) criterion.

2. Autoregressive Distributed Lag Model - ARDL

To estimate of relationship between the public revenues and the public expenditures, there is a need to define the functional form by stating the dependent and the independent variables. ARDL model is then created by adding the deterministic trend as well as the constant term based on the following equation [6]:

$$Y_t = a_0 + a_1 t + \sum_{i=1}^{p} \theta_i Y_{t-i} + \sum_{j=1}^{k} \sum_{I_j=0}^{q_j} B_{j,I_j} Z_{j,t-I_j} + \varepsilon_t \qquad (1)$$

Where, Y_t denotes the dependent variable (public expenditures), while Y_{t-i} represents its lagged values. Z_t denotes the independent variable (public revenues), whereas Z_{t-j} represents its lagged values. β_j denotes the coefficient of the independent variable. a_0 denotes the constant term of the model, while a_1 represents the trend coefficient. ε_t denotes the random error term. Finally, p and q denote the lag orders of the dependent and independent variables, respectively.

Once the functional form of the relationship has been specified, the model parameters are estimated by Ordinary Least Squares (OLS) method. Nonetheless, other possible optimal solutions are probably available that OLS cannot converge to, especially when there are nonlinearities in the relationship or when the classical conditions associated with the estimation residuals are violated. In this respect, the OLS estimates will be considered as initial values which are subsequently optimized using the assistance of computational intelligence algorithms to obtain more precise and more representative estimates of the relationship. Furthermore, this enhancement does not assume a specific probability distribution of model residuals, and thus it is more flexible to obtain the estimates using computational intelligence-based optimization in comparison with the traditional OLS estimation.

3. Computational intelligence

Computational Intelligence (CI) is a subfield of Artificial Intelligence (AI) and generally involves a multitude of processes of solving complicated problems in various settings [7]. CI deals with the producing mechanism of the solution, to produce an acceptable solution whereas AI deals with the results of producing mechanism that creates the best solution. Computational Intelligence encompasses a number of forms that include Evolutionary Computation (EC) [8], Artificial Neural Networks (ANN) [9], and Swarm Intelligence (SI) [10]. Evolutionary Computation and the related algorithms are regarded as some of the most rapidly developing techniques within the field of Computational Intelligence. These algorithms have been shown to be more efficient in solving optimization problems than other classical methods based on formal logic or mathematical programming [11]. Swarm Intelligence (SI) is considered one of the prominent subfields of Computational Intelligence (CI). This was first introduced by Wang and Beni [12], which is defined “any attempt to design algorithms or distributed problem-solving devices inspired by the collective behavior of social insect colonies and other animal societies” [13]. SI is a subfamily of meta-heuristic algorithms because SI algorithms simulate the behavior of natural systems and communities. They focus on the behavior of swarm individuals, their frequently complicated and extremely hierarchical lifestyles, and interactions between them, such as food sources, swarm protection, movement and navigation, nesting and colony building. Swarm Intelligence (SI) algorithms encompass a vast variety of algorithms, i.e., Ant Colony Optimization (ACO) [14], Particle Swarm Optimization (PSO) [15], Cuckoo Search (CS) [16], Krill Herd Algorithm (KH) [11], Firefly Algorithm (FA) [17], Artificial Bee Colony (ABC) [18], Dragonfly Algorithm (DA) [19], the Humpback Whale Algorithm (WOA) [20], Moth-Flame Optimization (MFO) [21], the Grey Wolf Optimizer (GWO) [22], and others. Any meta-heuristic algorithm is characterized by two primary elements exploitation and exploration, or intensification and diversification. Diversification is the creation of solutions that will exhaust the search space and intensive search narrows down to areas that give an indication that there is a good solution by exploiting information found in them. It is necessary to find a balance between intensification and diversification in order to avoid local optima and achieve global optimal solution [4].

3.1. Grey Wolf Optimization Algorithm (GWO)

3.1.1 Theoretical Background

The swarm intelligence algorithm that is regarded to be one of the most notable is The Grey Wolf Optimizer (GWO). Mirjalili et al. developed it in the year 2014 [22]. This algorithm resembles the hierarchy of the Grey Wolves and the collaborative hunting in the wild. Grey wolves are species of (Canis lupus) and they are categorized as predators which usually prefer to live in packs containing a mean population of between 5 and 12 wolves. Interestingly, this species has an exceptionally strict and dominant social structure where the members of the pack are classified into four categories, namely Alpha, Beta, Delta, and Omega. The first category is the Alpha group of leaders, consisting of males and females, who are mostly responsible for decisions related to hunting, sleeping locations, waking times, etc. The Alpha wolf is called the dominant wolf, and his orders are imposed on all members of the pack. The rest of the pack shows their obedience and agreement by lowering their tails. Interestingly, the Alpha wolf is not necessarily the strongest physically among the members of the pack, but he is the best in terms of management and organization, which means that the organization and discipline of the pack is much more important than his strength [23]. Beta group is the second layer in the social hierarchy and it acts as an advisor and subordinate to the Alpha wolves and help them in making decisions and other tasks they need to do in order to manage and organize the pack. Beta group can consist of both male and female. Moreover, the Beta wolf issues directives to the levels below and is believed to be the main candidate to be the head of the pack in case the Alpha wolf is so old or it passes away. The Delta class on the other hand is subordinate to the Alpha and Beta classes but dominant over the Omega class. The Delta class comprises scouts and guards whose role is to keep an eye on the frontiers of the territory, and to give warning to the pack, and also to keep it safe in case of the presence of any danger. It also involves the elders- seasoned wolves who once belonged to the Alpha or Beta classes whose duty is to support the hunters and feed the pack members. Also, the Delta class includes caregivers that assist wolves in the pack who are sick and injured. The last group of the grey wolves is the Omega group, which is the lowest in rank. This class must submit to the other three higher classes, its role is to calm the pack and take care of the pups. Members of this class are also the last ones allowed to eat. Although they are less important than the other groups, internal conflicts and problems have been observed in the event of the loss of this class [22]. The hierarchy of responsibility and distribution of the roles in the social order, as well as the implementation of the decisions of the Alpha wolf, contribute to the enhancement of the hunting effectiveness [24]. Hunting process involves a number of steps that follow a given sequence and are organized into tracking the prey, chasing it, Encircling it, harassing it, and finally attacking it [7].

3.1.2 Mathematical modeling

GWO algorithm uses the same mechanism pursued by the grey wolves in the leadership and role organization in the hunting process. A random group of grey wolves is formed and the pack members would be classified into four groups as mentioned above. The wolves are a candidate solution each, and the best three of these candidates are designated as Alpha, Beta and Delta respectively, with the rest of the candidate solutions being classified as Omega. The first three groups lead the hunting (optimization) process and the rest of the wolves follow them in the following steps [24, 25]:

1. **Encircling prey** The hunting behavior of gray wolves is to surround the prey first and to model mathematically such an encirclement behavior the following equation is established:

$$\vec{D} = \left| \vec{C} \,.\, \vec{X_p}(t) - \vec{X}(t) \right| \tag{2}$$

$$\vec{X}(t+1) = \vec{X_p}(t) - \vec{A}.\vec{D} \tag{3}$$

Where, D refers to the distance vector between the wolf and the prey. t denotes the current iteration. $\vec{X}(t)$ represents the position of the wolf at iteration t, while $\vec{X}_p(t)$ represents the position of the prey at iteration t. The vectors $\vec{A}$ and $\vec{C}$ are used to update the position of the wolf relative to the prey and are calculated using the following equations:

$$\vec{A} = 2\vec{a}\,.\,\vec{r_1} - \vec{a} \qquad , a = 2 - t\,(2/T) \tag{4}$$

$$\vec{C} = 2\,\vec{r_2} \tag{5}$$

Where (a) decreases linearly with the iterations, its value at the first iteration equals 2 and becomes zero at the final iteration. The terms (r_1, r_2) are random vectors with their values being in [0, 1] and (T) refers to maximum iteration. Based on the distance (D) and the extent calculated in equations (2) and (3), the candidate wolf $(X(t))$ can update its position (X(t+1)) and approach the prey X_p (the optimal solutions) with the help of the parameters (A, C), Figure 1 shows the approach mechanism of the grey wolf highlighted in yellow as it moves toward the prey.

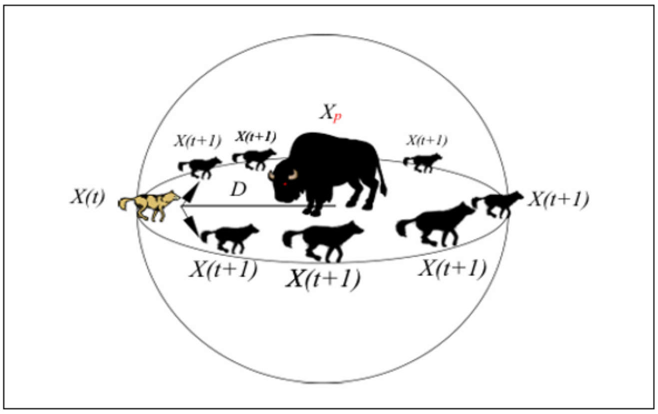


Figure 1: shows the potential locations that the grey wolf can assume relative to the prey depending on equations (2) and (3) [7].

2. **Hunting** The grey wolves possess the inherent skills of tracking their prey and this is normally facilitated by the Alpha wolf. This behavior is mathematically modeled with the help of equations Equation (6), Equation (7). As the optimum location of the prey is unknown, the best three candidate solutions namely Alpha, Beta, and Delta are chosen, since they have the most information about the location of the prey X_p. Even though multiple wolves (i.e. multiple solutions) can exist within a single category, to simplify the process, each category is represented by a single wolf or a single solution contained within it. As such the leading three wolves are kept and the fourth group (Omega) adjusts its position based on the quality candidate solution. Differently put, the Alpha, Beta and Delta wolves determine the location of the prey and the rest of the wolves update their locations at random locations about the prey. Figure 2 illustrates this.

$$\begin{aligned}
\vec{X}_1 &= \vec{X}_\alpha(t) - \vec{A}_1 \cdot \vec{D}_\alpha, & \vec{D}_\alpha &= \left|\vec{C}_1 \cdot \vec{X}_\alpha - \vec{X}\right|, \\
\vec{X}_2 &= \vec{X}_\beta(t) - \vec{A}_2 \cdot \vec{D}_\beta, & \vec{D}_\beta &= \left|\vec{C}_2 \cdot \vec{X}_\beta - \vec{X}\right|, \\
\vec{X}_3 &= \vec{X}_\delta(t) - \vec{A}_3 \cdot \vec{D}_\delta, & \vec{D}_\delta &= \left|\vec{C}_3 \cdot \vec{X}_\delta - \vec{X}\right|.
\end{aligned} \tag{6}$$

$$\vec{X}(t+1) = \frac{\vec{X}_1 + \vec{X}_2 + \vec{X}_3}{3} \tag{7}$$

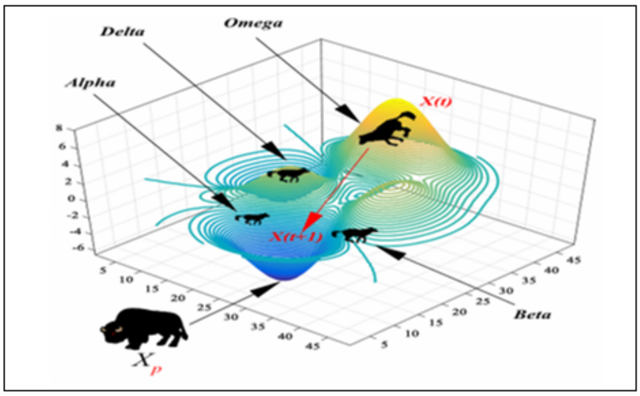


Figure 2: shows how the best solution is determined with the help of the best three solutions in GWO algorithm [7].

3. **Attacking prey (exploitatio)**

   Grey wolves accomplish this by hunting the prey after it has ceased movement. This step has the form of slowly reducing the values of $\vec{a}$ and $\vec{A}$ as the search progresses ($t \to T$). The A is a random value between $[-a, a]$, and when $|A| < 1$, this increases the wolves to attack the prey, which is termed as exploitation. On the other hand, when $|A| > 1$, the wolves go away of the prey.

   At the same time, C still produces random numbers in the range of $[0, 2]$ independent of the number of iterations, which increases the diversification of solutions produced by the algorithm and allows the wolves to find the prey. This is referred to as exploration. This is depicted in the following figure.

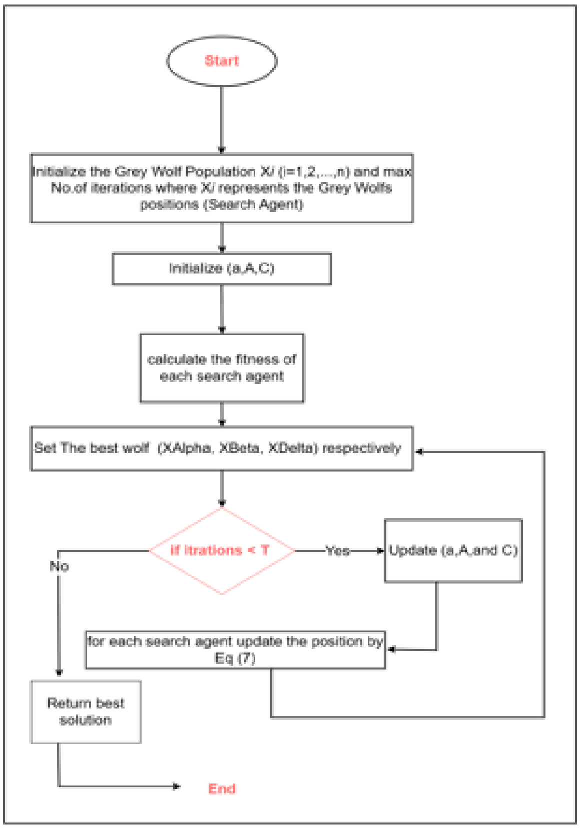


Figure 3: Flowchart depicts the operation principle of the (GWO) algorithm.

3.2. Whale Optimization Algorithm – WOA

3.2.1 Theoretical Background

The humpback whale is considered to be one of the seven key species among the whales and a member of the baleen whale group (Mysticetes). Whales have similar spindle cells in their brains as those of humans only that they are more in number. In addition, whales do not sleep but only half of the brain goes to sleep at a given time since they have to surface ensuring that they breathe after which they go back to the water. It has been demonstrated that whales can think, learn, form judgments, communicate, and have feelings just like humans, albeit to a lesser extent [26]. Whale Optimization Algorithm (WOA) is one of the Swarm Intelligence (SI) algorithms and can be considered as a meta-heuristic algorithm. Mirjalili and Lewis developed it in 2016 [20]. This algorithm is a simulation of the humpback whale hunting mechanism. The primary distinction to the GWO algorithm is that it models the hunting or food-search behavior whereby WOA uses a random search agent in order to find the prey (the best solution) and it is the candidate solution or the best search agent. It also circles the prey in a spiral way down to the top. In this motion, the humpback whale creates a bubble-net full of air as depicted in Figure 4. This action is aimed at causing a disturbance in the hunting ground, whereby the bubbles reduce the visibility of the prey, in assisting them to enclose the prey and avoid getting out of the bubble wall to set up to capture them. Interestingly, such a hunting technique is known as bubble-net feeding and it is a distinctive feeding behavior that is only practiced by the humpback whale [27]. In studying this behavior, scientists realized that it consists of two maneuvers that are related to the bubble-net. The former is referred to as upward spirals whereby the humpback whale dives to a depth of approximately 20 meters and then starts expelling bubbles as it goes up. The second maneuver is known as double loops, and is comprised of three stages; the coral loop, lobtail and the capture loop.

3.2.2 Mathematical modeling

Whale Optimization Algorithm (WOA) involves a starting point of the search or optimization process whereby an initial group of random solutions (whales) is selected in the feasible solution space with each whale being a candidate solution. The solutions will be subsequently updated on the basis of the objective function or optimization rules till a stopping criterion is achieved or the maximum number of iterations is achieved as illustrated in Figure 5. This algorithm has mathematical modeling, which can be split into three sequential steps:

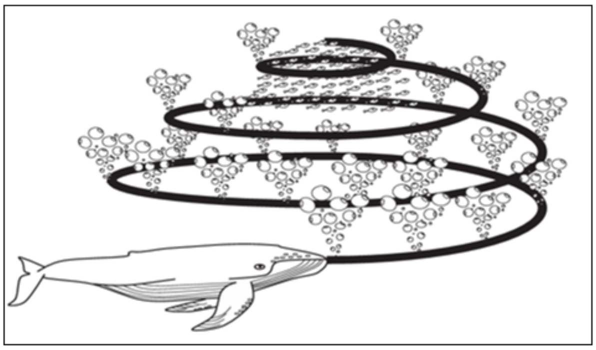


Figure 4: shows the feeding behavior of the humpback whale when feeding on the bubble-net [20].

1. **Encircling prey** the first stage of this algorithm involves wrapping around the prey. In the search space, the humpback whales are not aware of the best position to be when preying. Thus, such a step is modeled on the assumption that the targeted prey is the best solution that has ever been in place, and the whale nearest to the prey is the best search agent. The other search agents will then update their positions using the best search agent. The formula that mathematically describes this process is the following [20, 28]:

$$\vec{D} = \left| \vec{C} \cdot \vec{X}^*(t) - \vec{X}(t) \right| \tag{8}$$

$$\vec{X}(t+1) = \vec{X}^*(t) - \vec{A} \cdot \vec{D} \tag{9}$$

Where,

- D: Represents the distance between the search agent (whale) and the desired prey.
- (t): Current iteration.
- $\vec{X}^*(t)$: Refers to the optimal position of a solution and can be regarded as a local optimum.
- $\vec{X}_p(t)$: Represents the position of the prey at iteration (t).

$\vec{X}(t)$: Current search agent vector.

$\vec{X}(t+1)$: The new location vector of the current search agent.

$\vec{A}, \vec{C}$: Two coefficient vectors used to update the position of the whale toward the best local solution and are computed using the following equations.

$$\vec{A} = 2\vec{a}\,.\,\vec{r} - \vec{a} \qquad , a = 2 - t\,(2/T) \tag{10}$$

$$\vec{C} = 2\,\vec{r} \tag{11}$$

Where (a) decreases linearly with the iterations and its value at the first iteration as 2 and at the last iteration, it becomes zero. (r) is a random vector with their values being in $[0, 1]$ and (T) refers to maximum iteration. And the primary goal of equations Equation (10), Equation (11) is to reach a trade-off between exploration and exploitation.

2. **Bubble-Net Attacking** This stage of the feeding behavior is modeled based on the concepts of shrinking encircling and spiral position updating, as explained below.

   I. **Shrinking encircling mechanism:** This is done by varying the coefficient $(\vec{A})$ within the range $[-1, 1]$ and reducing the value of (a) linearly as the number of iterations increases as Equation (9). Consequently, the new position of the search agent will lie between its current position and the position of the optimal search agent.

   II. **Spiral position updating:** At this stage, every humpback whale calculates how far it is to the existing best whale and then travels in a spiral-like motion around it. The following equation can be used to model this process:

$$\begin{aligned} \vec{X}(t+1) &= \vec{D}'\,e^{bl}\cos(2\pi l) + \vec{X}^*(t) \\ \vec{D}' &= \left|\vec{X}^*(t) - \vec{X}(t)\right| \end{aligned} \tag{12}$$

   Where,

$\vec{D}'$: Refers to the distance between the whale and the prey (the best solution found so far).

b: A constant used to define the shape of the logarithmic spiral.

l: A random number in the interval $[-1, 1]$.

It has been observed that the humpback whales circle on the prey in a circular manner that narrows down as they go around in a spiraling fashion. In order to model such concomitant behavior, we take an equal probability of 0.50 of either the shrinking encircling mechanism or the spiral model in updating the positions of the whales during optimization, following formula:

$$\vec{X}(t+1) = \begin{cases} \vec{X}^*(t) - \vec{A}\cdot\vec{D}, & \text{if } p < 0.5, \\ \vec{D}'e^{bl}\cos(2\pi l) + \vec{X}^*(t), & \text{if } p \geq 0.5. \end{cases} \tag{13}$$

Where (p) a random value in the interval between $[-1, 1]$.

III. **Search for Prey (Exploration Phase):** the same strategy that relied on the adjustment of the value of the coefficient used during the exploitation stage can be pursued to improve the capacity of the algorithm to diversify the solutions, when $|A| \geq 1$, the humpback whales start searching randomly based on the location of the other whales which means that the search agent has to leave the local solution as opposed to the exploitation stage which happens when $|A| < 1$.

At the same time, will $\vec{C}$ continue producing random values in the range $[0, 2]$ irrespective of the iteration number and this operation increases the capability of the algorithm to execute a global search. The mathematical equation of this phase is written as following:

$$\begin{aligned} \vec{D} &= \vec{C}\cdot\vec{X}_{\text{rand}} - \vec{X}(t), \\ \vec{X}(t+1) &= \vec{X}_{\text{rand}} - \vec{A}\cdot\vec{D}. \end{aligned} \tag{14}$$

Where $\vec{X}_{\text{rand}}$ refers to a random position vector (a random whale) chosen among the whales conducting the search for the solution.

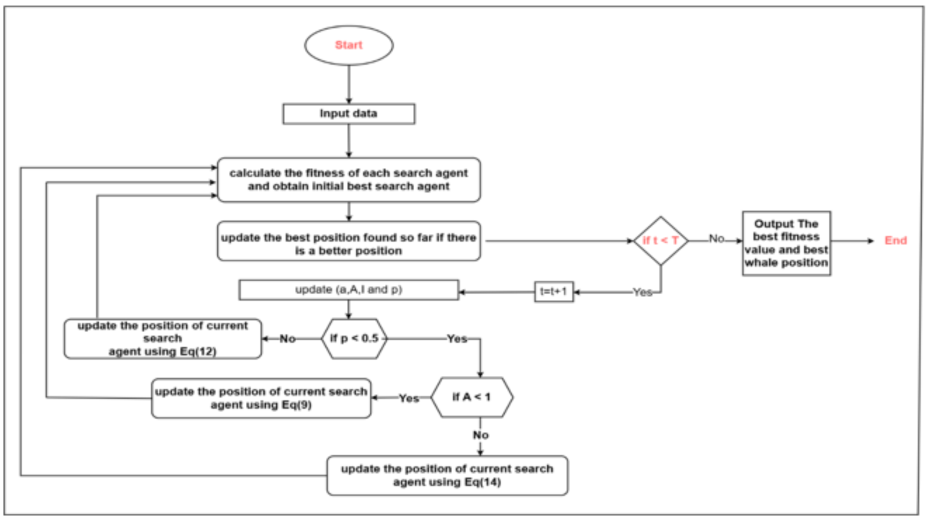


Figure 5: Flowchart depicting the working mechanism of the WOA algorithm.

4. Optimizing the parameters estimation

In this study, the parameter estimations of the proposed models are optimized by computational intelligence optimization algorithms, namely Grey Wolf Optimizer (GWO) and Whale Optimization Algorithm (WOA). The estimation process begins with the parameter estimates derived from the traditional ARDL model which are used as the initial solution. These estimates are then further improved with an iterative numerical search conducted by each optimization algorithm. An objective function with Huber loss function is used for optimization that offers a robust estimation framework by minimizing the impact of large prediction errors while preserving high efficiency for small errors. The mathematical expression of the objective function is shown below and the optimization procedure is the same as each algorithm [29].

$$(ObjectiveFunction) = \min\left(HuberLoss\right)$$
$$\mathrm{L}_{\delta}(\mathrm{e}) = \begin{cases} \frac{1}{2}e^2, & |e| \leq \delta \\ \delta\left(|e| - \frac{1}{2}\delta\right), & |e| > \delta \end{cases} \tag{15}$$

The proposed models use the Huber loss as the objective function in the optimization process, whereas the conventional Autoregressive Distributed Lag (ARDL) model uses the Mean Squared Error (MSE) as the objective function and estimates its parameters by minimizing the MSE, using the Ordinary Least Squares (OLS) method. The Huber loss is more efficient than the M-estimator and less affected by outliers, providing a more robust estimation framework. The optimization process starts from the estimated parameters from the conventional ARDL model, followed by the iteration of the parameters through the Grey Wolf Optimizer (GWO) and Whale Optimization Algorithm (WOA) to minimize the Huber loss and to get more robust parameters with better forecasting performance. In the evaluation stage, the forecasting performance of the models is assessed using the Root Mean Squared Error (RMSE) and Mean Absolute Error (MAE). In addition, the Huber loss values of the three models (ARDL-OLS, ARDL-GWO, and ARDL-WOA) are compared to evaluate the effectiveness of the optimization algorithms in minimizing the objective function. Furthermore, the models are statistically compared using the Diebold–Mariano (DM) test to determine whether the differences in forecasting accuracy are statistically significant [30]. The Directional Accuracy (DA) is also employed to evaluate each model's ability to correctly predict the direction of changes in the time series [31].

5. Empirical Results and Discussion

5.1. Data

This study is based on monthly data covering the period from October 2012 to April 2026, comprising a total of 163 observations. The data represent monthly flow values rather than cumulative stocks, reflecting the actual monthly flows of public expenditure as the dependent variable and public revenue as the independent variable. The data were obtained from the monthly statistical bulletins published on the official website of the Central Bank of Iraq.

To develop the proposed models and evaluate their performance, the dataset was chronologically divided into three subsets. The first 70% of the observations (114 observations) were allocated to the training set for model estimation. The subsequent 37 observations (approximately 23% of the dataset) were assigned to the validation set for hyperparameter tuning and model selection. Finally, the remaining 12 observations, representing one complete year (approximately 7% of the total dataset), were reserved as the test set for out-of-sample forecasting evaluation, with the aim of assessing the predictive ability of the models on unseen data.

5.2. ARDL Model Results

Since stationarity is a fundamental requirement for constructing time series models and dynamic regression models such as the ARDL model, the KPSS test was employed to examine the stationarity of the public revenues and public expenditure series.

Table 1: KPSS Unit Root Test Results

Variable	Level		First Difference		Order of Integration
	LM Statistic	Critical Value	LM Statistic	Critical Value	
Expenditure	1.191	0.463	0.226	0.463	I(1)
Revenues	0.272	0.463	–	–	I(0)

The results of KPSS unit root test are shown in Table (1). The results suggest that the KPSS test statistic (0.272) is less than the critical value (0.463), which suggests that the revenue series is stationary at the level, $I(0)$. The public expenditure series, on the other hand, is not stationary at the level as the test statistic (1.191) is greater than the critical value (0.463). Taking the first difference, however, the KPSS test statistic drops to 0.226, which is less than the critical value of 0.463, suggesting that the series is stationary at the first difference level, and thus is integrated of order one, $I(1)$. The results indicate that all the variables are stationary and meet the stationarity conditions of the ARDL model.

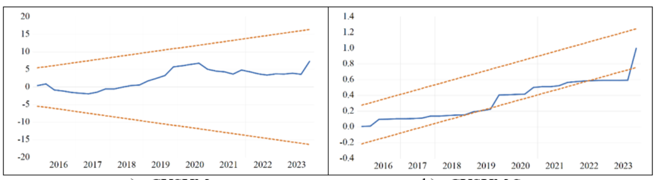
After establishing the order of integration of the study variables, the optimal lag structure of the Autoregressive Distributed Lag (ARDL) model was determined using the Akaike Information Criterion (AIC). A total of 72 alternative model specifications were evaluated, and the selection process identified the ARDL(1,4) specification with an intercept as the optimal model, as it yielded the lowest AIC value of 5.19. Accordingly, the ARDL(1,4) specification was selected for the subsequent estimation and analysis of both the short-run and long-run dynamic relationships among the study variables. The existence of a co-integration relationship between public revenues and public expenditure was also examined using the Bounds Test. The obtained F-statistic was about 56.85 which is above the upper-bound critical value at the 5 per cent level of significance. As a result, the null hypothesis will be rejected and the alternative one accepted, which will prove the existence of a long-run equilibrium relationship between the examined variables.

After confirming the existence of a long-run co-integration relationship among the study variables, an ARDL(1,4) model was estimated. The estimated model had an R^2 of 0.35, which means that the explanatory variables explain 35% of the variation in the dependent variable. In addition, the overall model was statistically significant $(F = 9.507; p < 0.001)$, indicating that the estimated coefficients for the model were significant together. A set of econometric diagnostic tests were performed to assess the model's econometric soundness, which are presented in Table (2). Breusch-Godfrey Serial Correlation LM test was used to test for serial correlation in the residuals, while Breusch-Pagan-Godfrey and White tests were used to test for heteroskedasticity. Normality of the residuals was checked using the Jarque-Bera test and the presence of possible functional form misspecification was tested using the Ramsey RESET test. The stability of the estimated coefficients was also assessed with the CUSUM and CUSUM of Squares (CUSUMSQ) tests, as shown in Figure 6.

Table 2: presents the results of the goodness-of-fit diagnostic tests

Diagnostic Test	Statistic	Value	p-value
Breusch–Godfrey LM	F-statistic	1.755	0.178
	LM ($T \times R^2$)	3.695	0.157
Ramsey RESET	F-statistic	13.698	< 0.001
	Likelihood ratio	13.861	< 0.001
Breusch–Pagan–Godfrey	F-statistic	0.945	0.466
	LM ($T \times R^2$)	5.740	0.452
	Scaled explained SS	39.435	0.0038
White Test	F-statistic	0.9457	0.5699
	LM ($T \times R^2$)	27.2114	0.4524
	Scaled explained SS	50.6007	< 0.001
Jarque–Bera Test	JB-statistic	995.270	< 0.001

The diagnostic test results reveal that the estimated ARDL model has most of the classical econometric assumptions. The Breusch-Godfrey test shows that there is no evidence of serial correlation in the residuals as the probability value is 0.157, which is greater than the 5% significance level. Similarly, the Breusch-Pagan-Godfrey and White tests provide probability values of 0.452, showing that there is no evidence of heteroskedasticity and that the variance of the error term is constant. But the Jarque-Bera test suggests that the residuals do not follow a normal distribution ($p < 0.001$). In addition, the Ramsey RESET test is highly significant ($p < 0.001$), indicating possible functional form misspecification or nonlinear relationships not well represented by the usual ARDL specification. As for parameter stability, the cumulative sum in the CUSUM test is within the 5% critical bounds over the sample period, which means that the model parameters are generally stable over time. The CUSUMSQ test, on the other hand, shows small deviations from the critical bounds, indicating that the parameters were not completely stable throughout the study period, but rather some minor structural changes occurred. Overall, these results suggest that the ARDL model, in its traditional form, is a good fit for the data, but that it may not be able to explain all of the nonlinearity in the data. Thus, the use of advanced metaheuristic optimization algorithms like Grey Wolf Optimizer (GWO) and Whale Optimization Algorithm (WOA) could help to improve the parameter estimation and the predictive performance of the ARDL model.



a) CUSUM b) CUSUM Square

Figure 6: illustrates the reliability test of the estimated model

5.3. ARDL-GWO and ARDL-WOA Model Results

The estimated coefficients of the conventional ARDL(1,4) model were used as the initial solution to the optimization process instead of random values. This approach ensures that the economically relevant parameter estimates from the classical econometric model are maintained while the metaheuristic algorithms are used to determine the values of the coefficients that provide better forecasting results without losing the economic meaning of the original model. The flexibility of the optimization problem was achieved by setting the bounds of each coefficient within 90% of the ARDL estimate, while ensuring a realistic search space. This range is a compromise between exploration and exploitation, in that the algorithms will explore to find better values for the parameters but not stray too far from the economically plausible estimates. Thus, the optimization process is more efficient and does not lead to unstable or economically unfeasible solutions. Three sets of data were used for optimization: training, validation and testing. The testing sample was only used for the final evaluation of the forecasting, while the training and validation samples were used during optimization process. The diagnostic tests of the conventional ARDL model showed that the residuals were not normally distributed and hence the conventional Mean Squared Error (MSE) was not used as the optimization objective but the Huber loss function was used. Huber loss has the best of both worlds of MSE and MAE; it is sensitive to small errors, yet it is not as strongly affected by large errors, especially when there are outliers or non-normal residuals. Moreover, the threshold parameter δ was not chosen by chance but calculated based on the residuals of the traditional ARDL model with the value of $1.345MAD$ (Median Absolute Deviation). This is a strong estimator that can automatically adjust the threshold to suit the dispersion of the data and yet is robust in the presence of outliers. The objective function was designed to be a weighted average of the Huber loss, with 20% weight on the training sample and 80% on the validation sample, to increase the generalization capability of the model and to avoid overfitting. This weighting scheme places more importance on the model's predictive power on new data, rather than just minimizing the training error, resulting in improved out-of-sample forecasting performance. The Ridge penalty with respect to the conventional ARDL coefficients was added to the objective function in order to maintain the economic properties of the original ARDL specification. This regularization will prevent the optimization algorithms from deviating too much from the initial econometric parameter estimates and thus prevent unrealistic parameter values while at the same time ensuring optimization stability and minimizing the risk of overfitting. Finally, a grid search was conducted to check the various combinations of the optimization parameters for Grey Wolf Optimizer (GWO) and Whale Optimization Algorithm (WOA). In this study, three population sizes $(100, 150, and 200)$ and three maximum number of iterations $(300, 500, and 1000)$ were tested for each algorithm, leading to nine sets of parameters per algorithm. The optimum solution that minimizes the objective function (Huber loss) was chosen and the optimized values of the coefficients were used to build the ARDL-GWO and ARDL-WOA models. The performance of the conventional ARDL model and the optimized ARDL-GWO and ARDL-WOA models was then evaluated based on the RMSE, MAE and the Directional Accuracy (DA) for both in-sample (training and validation) and out-of-sample (testing) periods. Moreover, the Diebold–Mariano (DM) test was used to evaluate if the forecasting accuracy gap of the competing models was statistically significant. The results shown in Table (3).

The comparison of forecasting performance between the conventional ARDL model and the optimized ARDL-GWO model and ARDL-WOA model is shown in Table (3) in terms of Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Directional Accuracy (DA), and the Diebold–Mariano (DM) test. The in-sample results show that the conventional ARDL model had the lowest values of RMSE (3.042) and MAE (1.920) among the optimized models. This result is not surprising since the parameters of the conventional ARDL model are obtained using the Ordinary Least Squares (OLS) method, which is a method in which the estimation error is minimized in the training sample.

Table 3: Comparison of Forecasting Performance among ARDL Models

Model	In-sample		Out-of-sample		Directional Accuracy (%)	Diebold–Mariano Test	
	RMSE	MAE	RMSE	MAE		DM Statistic	p-value
ARDL–OLS	3.042	1.920	4.634	3.8419	63.64	–	–
ARDL–GWO	3.324	2.081	4.118	3.0397	72.73	2.7880	0.0176
ARDL–WOA	3.328	2.085	4.113	3.0337	72.73	2.7677	0.0183
ARDL–GWO vs. ARDL–WOA			–			1.1228	0.2855

The optimized models, on the contrary, were clearly superior in the out-of-sample forecasting evaluation when compared to the conventional ARDL model. The RMSE for the conventional ARDL model was 4.634 whereas the RMSE of ARDL-GWO and ARDL-WOA models were 4.118 and 4.113 respectively, which represents improvement of 11.14% and 11.24% respectively. Similarly, the MAE improved from 3.8419 to 3.0397, and 3.0337, respectively, for the data that was not used to train the model, showing a significant improvement in forecasting accuracy for unseen data. Conventional ARDL model also showed an improvement in DA from 63.64% to 72.73% for ARDL-GWO and ARDL-WOA, respectively, which has led to an enhanced ability of these optimized models to predict the direction of the changes in public expenditure. To verify that these improvements were not due to random variation, the Diebold–Mariano (DM) test was employed to compare the forecasting accuracy of the competing models. The results revealed statistically significant differences between the conventional ARDL model and each optimized model (ARDL-GWO: $DM = 2.788, p = 0.0176$; ARDL-WOA: $DM = 2.7677, p = 0.0183$), confirming the superior forecasting performance of both optimized models at the 5% significance level. In contrast, no statistically significant difference was found between ARDL-GWO and ARDL-WOA ($DM = 1.1228, p = 0.2855$), indicating that the two optimization algorithms achieved comparable forecasting performance. As shown in Table (4), the objective function values further confirm the effectiveness of the optimization process. The objective function decreased from 6.593 for the conventional ARDL model to 5.788 for ARDL-GWO and 5.788 for ARDL-WOA. This indicates that both optimization algorithms successfully minimized the Huber Loss while maintaining parameter estimates close to those of the conventional ARDL model through the inclusion of a Ridge Penalty term in the objective function. Consequently, the optimized models achieved a better balance between goodness of fit and generalization ability, which was reflected in their improved out-of-sample forecasting performance.

Table 4: Objective Function (Huber Loss) Values of the Compared Models

Method	Objective Function (Huber Loss)
ARDL–OLS	6.593
ARDL–GWO	5.788
ARDL–WOA	5.788

Figure 7 illustrates the convergence behavior of the objective function during the optimization process for both the GWO and WOA algorithms. It can be observed that both algorithms converge rapidly toward stable objective function values, confirming the effectiveness and stability of the optimization process.
The results indicate that, from an economic viewpoint, the GWO algorithm and WOA were able to improve the performance of the ARDL model in capturing the dynamic relationship between public revenues and public expenditures, thus minimizing the forecasting errors and increasing the accuracy of forecasting the future changes. This finding is particularly important because public expenditure largely depends on the level of available public revenues. Thus, better forecasting of public spending will help to build more realistic government budgets, limit the risk of unforeseen budget deficits or surpluses, and increase the efficiency of financial planning and fiscal policymaking.

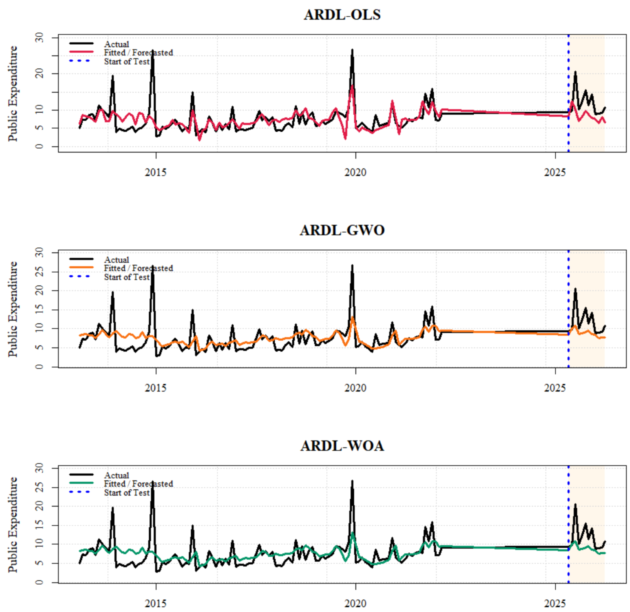


Figure 7: Actual and predicted public expenditures from the ARDL–OLS, ARDL–GWO, and ARDL–WOA models.

The results show that the traditional ARDL model combined with the metaheuristic optimization algorithms significantly enhanced the ability of the model to predict the out-of-sample data with high accuracy and the economic meaning of the model. Finally, the Diebold–Mariano test further confirms that these improvements are statistically significant, compared to the conventional ARDL model, while a lack of significance between ARDL-GWO and ARDL-WOA suggests that the two models have similar forecasting performance with ARDL-WOA marginally better in terms of out-of-sample forecasting errors. Hence, the ARDL-GWO and ARDL-WOA models are more efficient alternatives of the traditional ARDL model for economic forecasting applications.

6. Conclusion

Based on the results of this study, the combination of the Autoregressive Distributed Lag (ARDL) model with the Grey Wolf Optimizer (GWO) and Whale Optimization Algorithm (WOA) has significantly enhanced the out-of-sample forecasting capabilities over the traditional ARDL model. In particular, both optimized models had less RMSE and MAE values and higher Directional Accuracy (DA) values, showing that they better predicted the future values and the direction of changes in public expenditure. In addition, the Diebold–Mariano test results showed that the conventional ARDL model was statistically different from each of the optimized models, but there was no statistically significant difference between ARDL-GWO and ARDL-WOA, suggesting that both models performed the same in terms of forecasting performance, albeit with a descriptive benefit for ARDL-WOA because it had the lower out-of-sample forecasting errors. Furthermore, the results of the objective function also verify the effectiveness of the optimization process, as it provides a more balanced goodness of fit, model generalization and economic interpretability of the ARDL model.

The results have significant implications for economic and fiscal applications, because better forecasting of public revenues and expenditures leads to more accurate estimates for the preparation of the public budget, and improves the efficiency of fiscal planning and public resource management. Based on the results of this study, it is recommended that the ARDL models optimized by the metaheuristic optimization algorithms be used as decision support tools by the policy makers especially the Ministry of Finance in forecasting public revenues and expenditure, and in preparing future fiscal scenarios, as they are better in forecasting compared to the conventional ARDL model. The study also suggests the use of intelligent optimization methods in other econometric models and further research to compare the performance of other optimization algorithms or use the proposed methodology with other economic and financial variables to check the generalizability of the results and to increase the practical applications of the study.

Programming Code

The conventional Autoregressive Distributed Lag (ARDL) model was estimated using EViews software, the optimal lag length was determined, the required econometric and diagnostic tests were conducted, and the results of the estimation and forecasting were obtained. Subsequently, the R programming language was used to develop the proposed framework and implement the Grey Wolf Optimizer (GWO) and Whale Optimization Algorithm (WOA) for optimizing the parameters of the ARDL model. The optimization procedure, in-sample and out-of-sample forecasting, computation of forecasting performance measures, application of the Diebold–Mariano (DM) test, and figures illustrating the optimization process and forecasting results were also included in the R implementation. The entire programming code employed in this study is included in Appendix at the end of the report for the purposes of improving the repeatability of the results and for the purposes of application of the proposed methodology. The code also contains the data set employed in the empirical application, allowing for all parts of the application to be fully reproduced and the results reported to be checked without the need for external data sources. The code is designed to be used as a reference by researchers and practitioners who are interested in using, extending, or adapting the proposed framework for future research. https://github.com/suliamanhussien83-ux/ARDL-GWO-ARDL-WOA

Limitations of the Study

This study has some limitations. The empirical analysis was restricted to public revenues as the primary explanatory variable due to the limited availability of consistent monthly data for other macroeconomic variables. Important factors, such as international oil prices, the exchange rate, and inflation, were not included because reliable monthly data covering the entire study period were unavailable. The inclusion of these variables may further improve the explanatory and forecasting performance of the proposed models. Therefore, future research is encouraged to incorporate additional macroeconomic variables as comprehensive datasets become available.

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