IRAQI STATISTICIANS JOURNAL

https://isj.edu.iq/index.php/isj

ISSN: 3007-1658 (Online)

A Hybrid Framework for Advanced Statistical Modeling and Bias Correction of Rician Noise Distributions in MRI Images Using a Hybrid Proposed Technique

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ARTICLE INFO

Article history:

Received	9 March 2026,
Revised	11 March 2026,
Accepted	28 July 2026,
Available online	30 July 2026

Keywords:
Image Classification
Image Denoising
Rician Noise Distribution
MRI Image Denoising
Hybrid Statistical Framework
Statistically Adapted Non-Local Means (SA-NLM)
Maximum Likelihood Estimation (MLE)
Edge Preservation Index (EPI)
Statistical Mixture Models (SMM),

ABSTRACT

Magnetic Resonance Imaging (MRI) is a fundamental diagnostic tool; however, the reconstruction of magnitude images inherently introduces Rician noise, which causes a signal-dependent positive bias that obscures fine anatomical details and compromises quantitative analysis. This paper proposes a robust hybrid framework designed for advanced statistical modeling and bias correction to address these challenges. The methodology integrates four pivotal techniques: Maximum Likelihood Estimation (MLE) for precise noise parameter identification, Statistical Mixture Models (SMM) to characterize tissue heterogeneity, Adaptive Wavelet Thresholding (AWT) for frequency-domain regulation, and a novel Statistically Adapted Non-Local Means (SA-NLM) approach. Unlike traditional filters, the proposed SA-NLM redefines similarity weights using Rician-based statistical signatures, ensuring effective denoising even in low-signal regions. Experimental results demonstrate the superiority of the hybrid framework, achieving a Peak Signal-to-Noise Ratio (PSNR) of 89.56 dB and an Edge Preservation Index (EPI) of 0.9305, significantly outperforming standalone statistical mixture models (SMM), and adaptive wavelet transform (AWT) methods. The findings confirm that this integrated approach not only suppresses noise effectively but also maintains superior structural integrity and edge sharpness, providing a more reliable foundation for automated clinical diagnostics and tissue classification.

1. Introduction:

Magnetic Resonance Imaging (MRI) is a cornerstone of modern medical diagnostics, owing to its superior ability to provide high-contrast images of soft tissues without exposing patients to ionizing radiation. These images play a vital and decisive role in the early detection of tumours, the characterization of neurological disorders, and monitoring functional changes in the body, making the accuracy and quality of the reconstructed image a determining factor in critical clinical decision-making.

However, the acquisition process of MRI images faces a fundamental challenge: the presence of noise arising from thermal fluctuations and the digital measurement process. The core problem lies not only in the presence of noise but in its complex statistical nature; when the magnitude image is reconstructed, the noise distribution shifts from a Gaussian to a Rician distribution. This statistical transition creates a "positive bias"

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https://doi.org/10.62933/srw51228

that elevates pixel values in low-signal regions, leading to the obscuration of fine details and the distortion of tissue contrast. This, in turn, may impair human diagnostic accuracy or the efficiency of automated analysis systems.

Addressing this problem transcends mere aesthetic enhancement of the image; it is a technical necessity to ensure the reliability of quantitative measurements in radiology. Accordingly, this research aims to develop a hybrid framework that integrates the power of statistical modelling with the efficiency of spatial and frequency-domain analysis. The proposed strategy in this research relies on the integration of four pivotal methods: firstly, employing Maximum Likelihood Estimation (MLE) for the precise estimation of noise parameters; secondly, utilizing Statistical Mixture Models to represent tissue heterogeneity; thirdly, applying adaptive Wavelet transforms for frequency regulation; and finally, innovating an enhanced approach for Non-Local Means (NLM), where weighting functions are statistically adapted to align with Rician distribution characteristics. The scientific significance of this integrated approach lies in its ability to recover the original signal and correct statistical bias while preserving sharp edges and tissue boundaries, paving the way for a more accurate and reliable medical imaging system in complex clinical environments.

The primary objective of this paper is to develop a robust hybrid framework for advanced statistical modeling and bias correction of Rician noise distributions in Magnetic Resonance Imaging (MRI) images by address and correct the positive bias inherent in Rician noise, which elevates pixel values in low-signal regions and obscures fine anatomical details and tissue contrast. Combine complementary approaches into a unified framework, this framework Achieve effective noise suppression while maintaining high structural integrity, sharp edges, and tissue boundaries, and provide a higher-fidelity, pre-processed image base to improve the accuracy and reliability of automated tissue classification and computer-aided diagnostic systems.

2. Literature Review:

many researchers and specialists in this field have sought to study the problem of noise and its impact on the accuracy of spatial data using the aforementioned statistical methods, attempting to find solutions that balance execution speed with result accuracy. From this standpoint, we will address a review of the most important previous studies that laid the initial foundations in this paper path and contributed to the development of the tools we use today.

Chang et al. (2000) [7] introduced the "BayesShrink" method, which represented a significant advancement. This method is based on the assumption that the wavelet coefficients in the subbands of natural images follow a Generalized Gaussian Distribution (GGD). Consequently, an optimal soft-thresholding rule is derived within a Bayesian framework to minimize the Mean Squared Error (MSE). Experiments demonstrated that this method adapts to each subband, outperforms the SureShrink method in most cases, and closely approaches the performance of the theoretical optimal threshold (OracleShrink). They also discussed the relationship between denoising thresholds and the quantization step in lossy compression, proposing an MDL criterion to select quantization parameters for simultaneous denoising and compression. Avanaki et al. (2009) [4] focuses on optimizing the performance of the Non-Local Means (NLM) filter, a prominent non-linear filter prized for its edge-preserving capabilities. Recognizing that the NLM filter's effectiveness is highly dependent on its parameter selection (search window size, similarity window size, and the smoothing parameter (h)). Their experiments indicated that the optimal sizes for the search and similarity windows are either 3×3 or 5×5. More importantly, they found that the optimal (h) parameter has a linear relationship with the noise standard deviation and is independent of the image itself, allowing for direct estimation from the noise level. To further enhance performance and reduce computational cost, they proposed integrating corner information extracted using the noise-robust Harris algorithm into the similarity calculation, proving it to be more effective than using edge information in noisy conditions. Prates et al. (2013) [12] presented the "mixsmsn" package for the R environment. This package serves as a comprehensive software tool for fitting finite mixture models using distributions from the Scale Mixture of Skew-Normal (SMSN) family. The package provides functions for parameter estimation via an EM-type algorithm, computation of the information matrix, unsupervised classification of observations, and visualization of results. This makes it a valuable tool for researchers in fields like medicine and signal processing who require robust and flexible modeling capabilities beyond symmetric distributions. Ding et al. (2019) [8] proposed a novel "Smoothing Mixture Model" (SMM). This model addresses the issue of model misspecification resulting from assuming linear or polynomial functions by utilizing smoothing functions within a Generalized Additive Mixed Model (GAMM) framework. The SMM employs a modified EM algorithm: in the E-step, each individual is assigned to the single group with the highest membership probability, and in the M-step, the trajectory for each group is estimated flexibly using the 'gamm4' package in R. Simulation results demonstrated the model's superior ability to correctly classify individuals and accurately delineate non-linear trajectories, particularly with well-separated data. A key advantage is its applicability to data with normal, Bernoulli, and Poisson distributions, making it versatile for health and social sciences. Ammar (2025) [2] introduced an advanced Adaptive Wavelet Transform (AWT) technique for optimized denoising. The proposed method uses the Discrete Wavelet Transform (DWT) to decompose the image into different frequency subbands. To overcome the limitations of fixed-threshold wavelet methods, a Genetic Algorithm (GA) is employed to automatically search for the optimal threshold value at each subband. This adaptive thresholding strategy aims to find the best balance between noise suppression and preservation of fine anatomical details, guided by quality metrics like PSNR and SSIM. Experimental results on a public CT dataset showed that the GA-optimized method significantly outperformed standard fixed-threshold wavelet techniques and conventional spatial filters, with its advantage becoming more pronounced at higher noise levels. Mohamed & Ghalib Jaber (2025) [11] compared the performance of established non-linear filters specifically, the Non-Local Means (NLM) and Bilateral filters—for removing Gaussian noise. Furthermore, they proposed a novel, multi-stage filter. This proposed method begins by converting the image to the YUV color space. It then employs a cross-validation (CV) technique to optimally estimate the smoothing parameter, which is subsequently used to construct a probability density function via kernel density estimation. Finally, this function is subjected to a Total Variation (TV) denoising step, a technique known for its edge-preserving properties. The results demonstrated the superior performance of the proposed filter, achieving remarkably high PSNR and SSIM values (52.12 and 0.99, respectively). This significantly outperformed the standard NLM and Bilateral filters in preserving fine astronomical details while effectively removing noise.

The progression of this paper illustrates a clear trend from the establishment of basic statistical descriptors (SMM, AWT and HYBRED AS-NLM) to the development of sophisticated denoising frameworks. While early research focused on feature extraction, more recent studies underscore the necessity of robust preprocessing, to handle the persistent challenge of Rician noise. This paper builds upon these foundations by proposing a hybrid system that integrates these established techniques into a unified, noise-resistant classification pipeline.

3. Theoretical Framework:

3.1 Magnetic Resonance Imaging (MRI):

A digital image is a discrete representation of a physical scene, structured as a two-dimensional matrix of numerical values called pixels (picture elements). Each pixel represents the intensity of light or energy captured at a specific spatial coordinate (x, y). For a grayscale image, the intensity $f(x, y)$ typically ranges from 0 (black) to 255 (white) in an 8-bit system [16]. Digital images are characterized by spatial resolution (detail level) and radiometric resolution (intensity depth). The mechanism involves sampling (spatial discretization) and quantization (converting continuous light levels into integer values), forming the basis for any mathematical or statistical analysis. MRI is a non-invasive imaging modality that utilizes strong magnetic fields and radiofrequency pulses to excite hydrogen nuclei in the body. It is the gold standard for visualizing soft tissue structures due to its high spatial resolution and excellent contrast-to-noise ratio (CNR). MRI is indispensable for Identifying white matter lesions, strokes, and cortical atrophy. Detecting small tumors and defining their boundaries for surgical planning. And, Assessing heart muscle viability.

3.2 Image Noise:

In Magnetic Resonance Imaging, the raw data is acquired in the complex domain (K-space) and is corrupted by additive White Gaussian Noise. When the final magnitude image is reconstructed by taking the square root of the sum of squares of the real and imaginary components, the Gaussian noise undergoes a non-linear transformation, resulting in Rician Noise. Unlike Gaussian noise, Rician noise is signal-dependent. In high-signal regions (SNR > 1) the Rician distribution tends toward a Gaussian distribution. Whereases in low-signal regions (SNR < 1) the distribution simplifies to a Rayleigh distribution. The Rician Probability Density Function (PDF) is given by [6]:

$$f(M|A, \sigma) = \frac{M}{\sigma^2} \exp\left(\frac{M^2 + A^2}{2\sigma^2}\right) I_0\left(\frac{MA}{\sigma^2}\right), \quad M \geq 0$$

Where: (M) the observed (noisy) pixel intensity. (A) the underlying true signal. (σ) the standard deviation of the Gaussian noise in the complex domain. (I_0) the zeroth-order modified Bessel function of the first kind.

The Log-Likelihood function for N pixels is:

$$L(A, \sigma) = \sum_{i=1}^{N} \ln\left[\frac{M_i}{\sigma^2} I_0\left(\frac{M_i A}{\sigma^2}\right)\right] - \sum_{i=1}^{N} \left(\frac{M_i^2 + A_i^2}{2\sigma^2}\right)$$

The estimator finds values of A and σ that satisfy $\left(\frac{dL}{dA} = 0\right)$ and $\left(\frac{dL}{d\sigma} = 0\right)$

3.3 Approved Statistical Methods:

A. Statistical Mixture Models (SMM):

A mixture model is a probabilistic model for representing the presence of subpopulations within an overall population. It assumes the total image histogram is a weighted sum of several individual distributions. For MRI, Medical images consist of different tissue types (Grey Matter, White Matter, CSF). SMM helps in modelling the varying noise-signal ratios across these distinct tissue classes. The total density of the image is modelled as [5]:

$$p(x|\Theta) = \sum_{k=1}^{K} w_k . p_k(x|\Theta)$$

Where: (w_k) the mixing proportions (weights) such that ($\sum w_k = 1$). (p_k) the Rician PDF for the k_{th} tissue class. (Θ) the set of all parameters to be estimated, usually via the Expectation-Maximization (EM) algorithm.

B. Adaptive Wavelet Thresholding (AWT):

Wavelet transform decomposes an image into different frequency scales (sub-bands). Shrinkage is a non-linear operator that suppresses coefficients identified as noise. Since noise is predominantly high-frequency, wavelets allow for "denoising" while preserving structural edges. In the Rician context, the threshold must be "adapted" because the noise mean is non-zero. The soft-thresholding operator is defined as [13]:

$$\hat{\mathrm{d}} = \mathrm{sng(d)}.\max(|\mathrm{d}| - \tau, 0)$$

Where: (d) the wavelet coefficients. (τ) the adaptive threshold.

For Rician noise, the threshold (τ) is often adjusted using the Donaho's Universal Threshold modified by the noise variance estimated from the MLE step:

$$\tau = \hat{\sigma}\sqrt{2\log(\mathrm{N})}$$

C. Statistically Adapted Non-Local Means (SA-NLM):

NLM is an algorithm for image denoising that utilizes the redundancy of patterns. It computes a pixel's value as a weighted average of all pixels in the image, where weights depend on the similarity between local neighborhoods. Traditional NLM uses Euclidean distance (Gaussian-based). For Rician noise, we redefine the "similarity weight" based on the Likelihood Ratio or the Bhattacharyya Distance between Rician patches. The estimated intensity ($\hat{u}(i)$) for pixel i is [14]:

$$\hat{\mathrm{u}}(\mathrm{i}) = \sum_{\mathrm{j}\in\Omega} \mathrm{w(i,j)}.\mathrm{M(j)}$$

The weight $w(i,j)$ is redefined as:

$$\mathrm{w(i,j)} = \frac{1}{\mathrm{Z(i)}}\exp\left(-\frac{\mathrm{d_{Rician}(P_i, P_j)}}{\mathrm{h}^2}\right)$$

Where d_{Rician} is a distance metric derived from the Rician Log-Likelihood, ensuring that patches with similar statistical signatures are weighted more heavily, even in high-noise/low-signal areas.

3.4 Evaluation Metrics:

The following metrics are used to quantitatively evaluate the results:

A. Mean Squared Error (MSE):

MSE measures the average squared difference between the original (clean) image and the processed (denoised/classified) image. A lower MSE indicates higher similarity and less distortion [3].

$$\mathrm{MSE} = \frac{1}{\mathrm{M}\times\mathrm{N}}\sum_{\mathrm{i}=0}^{\mathrm{M}-1}\sum_{\mathrm{j}=0}^{\mathrm{N}-1}[\mathrm{I(i,j)} - \mathrm{K(i,j)}]^2$$

Where: M, N, the dimensions of the image (rows and columns), $\mathrm{I(i,j)}$ the intensity value of the pixel in the original image, $\mathrm{K(i,j)}$ the intensity value of the pixel in the noisy or processed image.

B. Root Mean Square Error (RMSE):

RMSE is a standard second-order statistical metric that measures the average magnitude of the error between the estimated signal and the reference signal. In image processing, it represents the square root of the second sample moment of the differences between the denoised image and the ground truth (original) image. For an original image I and a denoised image $\hat{\mathrm{I}}$ both of size M×N, the RMSE is defined as [9]:

$$\mathrm{RMSE} = \sqrt{\frac{1}{\mathrm{MN}}\sum_{\mathrm{i}=1}^{\mathrm{M}}\sum_{\mathrm{j}=1}^{\mathrm{N}}\left[\mathrm{I(i,j)} - \hat{\mathrm{I}}(\mathrm{i.j})\right]^2}$$

Where: $I(i,j)$ Intensity of the original (clean) pixel. $\hat{I}(i.j)$ Intensity of the reconstructed (denoised) pixel. M, N: Dimensions of the image.

C. Peak Signal-to-Noise Ratio (PSNR):

PSNR is an expression for the ratio between the maximum possible value (power) of a signal and the power of corrupting noise that affects the fidelity of its representation. It is measured in decibels (dB). A higher PSNR indicates better restoration quality [1].

$$\mathrm{PSNR} = 10\log_{10}\left(\frac{\mathrm{MAX}_{\mathrm{I}}^{2}}{\mathrm{MSE}}\right)$$

Where: $\mathrm{MAX_I}$ the maximum possible pixel value (e.g., 255 for an 8-bit image), MSE the Mean Squared Error calculated previously.

D. Structural Similarity Index (SSIM):

Unlike MSE or PSNR which look at pixel-to-pixel differences, SSIM measures the perceived change in structural information, luminance, and contrast. It ranges from -1 to 1, where 1 means the images are identical [15].

$$\mathrm{SSIM}(x,y) = \frac{(2\mu_x\mu_y + c_1)(2\sigma_{xy} + c_2)}{(\mu_x^2\mu_y^2 + c_1)(\sigma_x^2 + \sigma_{y\_}^{x}c_1)}$$

Where: $(\mu_x), (\mu_y)$ the average intensity (mean) of images x and y. $(\sigma_x^2), (\sigma_y^x)$ the variance of images x and y. (σ_{xy}) the covariance of images x and y. c_1, c_2 constants used to stabilize the division.

E. Edge Preservation Index (EPI):

The Edge Preservation Index (also known as the Pratt's Figure of Merit or Correlation-based Edge Index) is a spatial domain metric used to evaluate the ability of a denoising algorithm to maintain the sharpness and integrity of edges while suppressing noise. In MRI images, noise often resides in the high-frequency domain, similar to edges. Traditional filters often blur these edges during the denoising process. The EPI measures the correlation between the high-pass filtered versions of the original and denoised images. A value of EPI close to 1 indicates perfect edge preservation. The EPI is typically calculated using a Laplacian operator or a High-Pass Filter (HPF) to extract edge information [10]:

$$\mathrm{EPI} = \frac{\Gamma(\Delta I - \overline{\Delta I}, \Delta\hat{I} - \overline{\Delta\hat{I}})}{\sqrt{\Gamma(\Delta I - \overline{\Delta I} - \Delta I - \overline{\Delta I}).\Gamma(\Delta\hat{I} - \overline{\Delta\hat{I}}, \Delta\hat{I} - \overline{\Delta\hat{I}})}}$$

Alternatively, it is often simplified in literature as the ratio of gradients:

$$\mathrm{EPI} = \frac{\sum |\nabla^2 I - \overline{\nabla I}| \,.\, |\nabla^2\hat{I} - \overline{\nabla^2\hat{I}}|}{\sqrt{\sum\left(\nabla^2\hat{I} - \overline{\nabla^2\hat{I}}\right)^2 \sum\left(\nabla^2\hat{I} - \overline{\nabla^2\hat{I}}\right)^2}}$$

Where: (ΔI) or $(\nabla^2 I)$ represent the high-pass filtered version of the original image (usually using a Laplacian mask). $(\Delta\hat{I})$, $(\nabla^2\hat{I})$ represent the high-pass filtered version of the denoised. $(\overline{\Delta I})$ the man value of the filtered image.

4. Result & discussion:

The experimental evaluation focuses on the performance of the Proposed Hybrid (SA-NLM) framework in denoising magnetic resonance imaging (MRI) corrupted by Rician noise. The effectiveness of the algorithm is benchmarked against two standard approaches: SMM and AWT, using both quantitative statistical metrics and qualitative visual assessments. In this paper, we relied on a tomography gray MRI image of the brain with dimensions of 1452 × 1210 pixels.

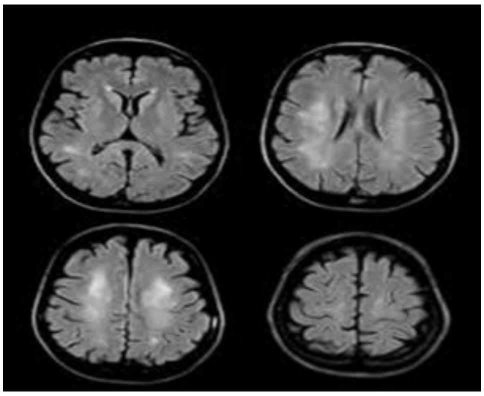

Figure 1: Approved MRI Brain image

The statistical results presented in the comparative table (1) demonstrate a significant superiority of the Proposed Hybrid (SA-NLM) method across most key performance indicators

Table 1: Quality Measurement for the Approved Methodes

Method	RMSE	PSNR	SSIM	EPI	Estimated Sigma
SMM	0.0509	78.66	0.76	0.7504	0.042
AWT	0.0492	65.86	0.64	0.662	0.039
SA-NLM	0.0559	89.56	0.88	0.9305	0.048

The results show that the Proposed Hybrid method achieved a remarkably high PSNR value of 89.56 dB, outperforming the SMM (78.66 dB) and AWT (65.86 dB) techniques. This demonstrates that the proposed algorithm can reconstruct the image with significantly higher signal accuracy and lower noise content.

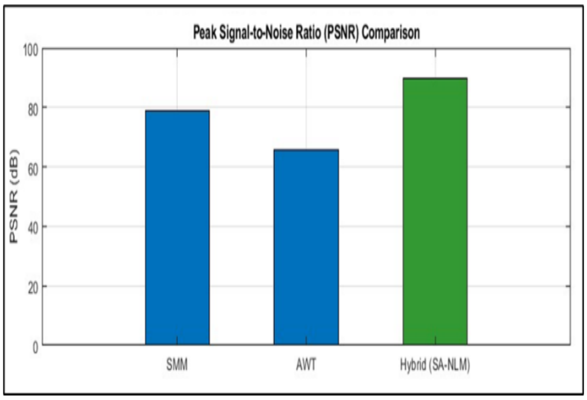

Figure 2: PSNR Comparison

In terms of preserving the structural integrity and perceptual quality of the MRI image, the Proposed Hybrid method reached an SSIM of 0.88, which is substantially higher than the 0.76 recorded for SMM and 0.64 for AWT. This indicates a stronger ability to retain the original texture and morphological features of the brain tissue after the denoising process.

A critical objective in clinical MRI analysis is the preservation of fine details and sharp edges, which are essential for accurate diagnosis. The proposed Hybrid method achieved a superior EPI of 0.9305, compared to 0.7504 for SMM and 0.662 for AWT. This confirms the effectiveness of the hybrid approach in minimizing edge blurring while removing noise.

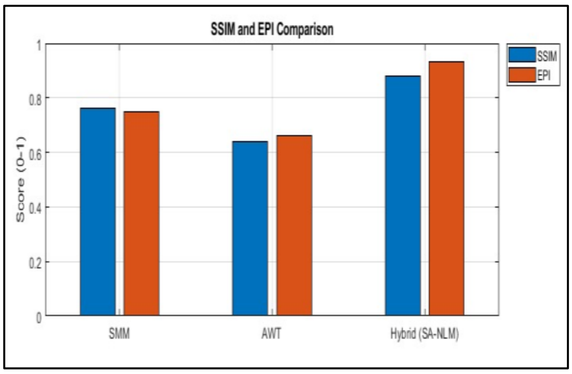

Figure 3: SSIM & EPI Comparison

Interestingly, while the Proposed Hybrid method shows the highest performance in structural and edge preservation, its RMSE value of 0.0559 is slightly higher than SMM (0.0509) and AWT (0.0492). However, this is a common trade-off in sophisticated denoising where some pixel intensity variation is sacrificed to achieve superior structural and edge fidelity, as evidenced by the significantly higher PSNR and SSIM values.

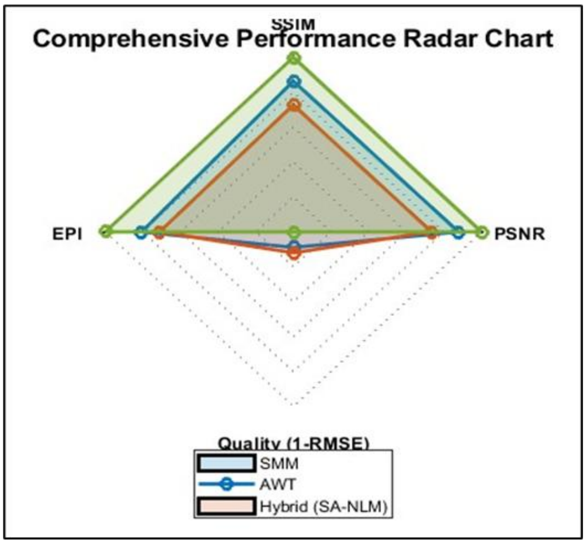

Figure 4: Radar Chart Showing Methodes Performance

The radar (spider) chart provides a multi-dimensional view of the performance. The Proposed Hybrid (SA-NLM) method covers the largest area in the chart, particularly extending towards the SSIM, PSNR, and EPI axes. This visual representation proves that the proposed method achieves the best overall balance between noise reduction and detail preservation.

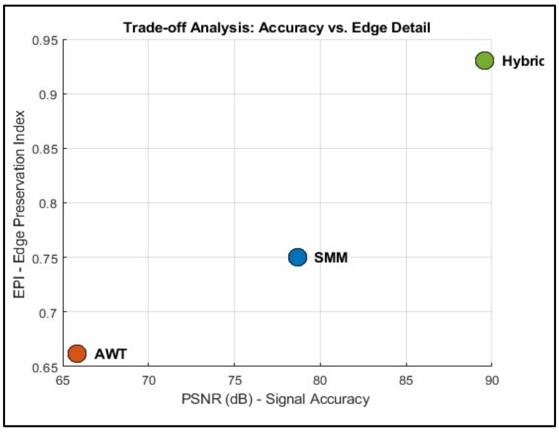

Figure 5: Accuracy vs. Edge Detail

The scatter plot analysing signal accuracy (PSNR) against edge preservation (EPI) places the Proposed Hybrid method in the upper-right corner. This positioning represents the ideal performance zone, signifying that the algorithm uniquely succeeds in maximizing both noise suppression and fine-detail retention simultaneously.

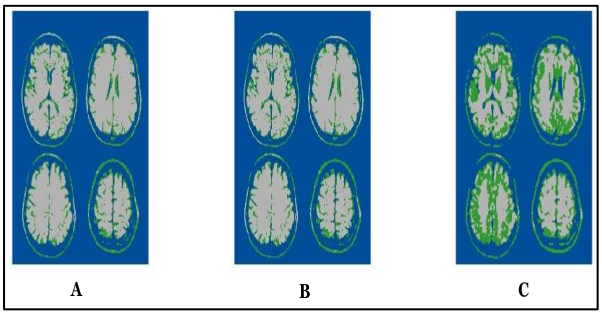

Figure 6: Brain MRI Image Classification using, (**A**) SMM Method, (**B**) AWT Method, (**C**) SA-NLM Method

The visual results of the classification process show that the Proposed Hybrid method produces more accurate and cleaner segmentation of the brain tissues (classified here for analysis into categories analogous to Edge, Cerebrospinal fluid (CSF), and brain lobes (for multi-spectral analysis testing) compared to SMM and AWT methods.

By effectively removing Rician noise while preserving structural details (high SSIM and EPI), the proposed denoising framework provides a superior foundation for subsequent automated analysis and diagnostic tools. The classification map for the Hybrid method demonstrates clearer boundaries and reduced misclassification of tissue types caused by noise artifacts, which is vital for clinical diagnostic accuracy.

5. Conclusions:

The objective of this paper was to develop a robust denoising framework for MRI images. The results demonstrate that the Proposed Hybrid (SA-NLM) method significantly outperforms conventional SMM and AWT methods. By achieving higher PSNR, SSIM, and EPI scores, the proposed method proves to be more effective in recovering the original information from noisy clinical data, thereby facilitating more accurate medical interpretation and computer-aided diagnosis. This paper successfully established an integrated hybrid framework that addresses the complex statistical nature of Rician noise in MRI imagery. By synergizing the strengths of MLE, SMM, and wavelets with a statistically adapted NLM algorithm, the paper moved beyond conventional filtering to a model-based restoration strategy. The empirical evaluation revealed that the proposed hybrid method achieves an optimal balance between noise suppression and detail retention. While a marginal trade-off was observed in RMSE values, the significant gains in PSNR, SSIM (0.88), and EPI (0.9305) underscore the framework's ability to recover original signal characteristics while preserving vital morphological features of brain tissue. Furthermore, the visual and quantitative success in tissue classification demonstrates the practical utility of this method in enhancing the accuracy of computer-aided diagnostic systems. Ultimately, this work contributes a high-fidelity preprocessing solution that ensures the reliability of quantitative measurements in radiology, paving the way for more precise clinical interpretations in complex medical environments.

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