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The Evolution of Estimation Methods and Applications of the Fama-MacBeth Model : A Review

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ABSTRACT

This literary review offers a comprehensive analysis of the evolution of methods for estimating and using the Fama-MacBeth regression model in research. The paper traces the model's development from a simple tool for testing the Capital Asset Pricing Model (CAPM) to a complex statistical framework used to analyze multifactor models and complex phenomena.
The review focuses on the key developments in the uses of the Fama-MacBeth model and its estimation. It also highlights the expansion of the model's applications.
In conclusion, this review shows that the Fama-MacBeth model remains a vital and effective tool in financial research, and its methodological and applied evolution has made it more powerful and flexible in facing the challenges of analyzing complex financial data.

1. Introduction

The Fama-MacBeth regression model is a statistical tool specifically designed for cross-sectional regression analysis, a powerful technique for studying relationships between variables across a set of units at a specific point in time. Cross-sectional regression aims to understand how the values of the dependent variable vary across these units based on the values of the independent variables.

The Fama-MacBeth model is used to identify the factors that most influence a given variable, evaluate the validity of models, and estimate model parameters that vary over time, especially when using proxy variables to represent factors that are difficult to measure directly. This model features a two-step approach:

The first step involves running a time-series regression on each unit to analyze the effect of the independent variables over time.

The second step uses the results of the first step and applies a cross-sectional regression to estimate the relationship between variables across all units at a given time period.

However, estimating this model faces several challenges. Although ordinary least squares (OLS) is traditionally used, it may not be the optimal tool when there are nonlinear relationships, when model parameters change over time, or when there are alternative variables or even missing values in the data.

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This review aims to comprehensively survey the recent research literature related to the uses and applications of the Fama-MacBeth regression model and its estimation methods.

2. Fama-MacBeth model

Methodologically, the Fama-MacBeth model assumes that the coefficients of the relationship between variables are constant across cross-sectional units at each given time period. These coefficients are traditionally estimated using the ordinary least squares method (Baillie et al., 2022)[12]. The model consists of two main steps, with the outputs from the first step becoming inputs for the second step

The Fama Macbeth regression method consists of two steps, **The first step** is a time-series regression that could be used to get time varying parameters of this regression in step 1, or to get a proxy variables which utilized it as an independent variable in the second step (cross-section regression) [7].

Suppose we have N dependent variables with k factors affecting the dependent variable.

$$Y_{i,t} = B_{0,i} + B_{1,i}X_{1,t} + B_{2,i}X_{2,t} + \cdots + B_{k,i}X_{k,t} + \varepsilon_{i,t} \quad \dots(1)$$

Where, $Y_{i,t}$ shows the dependent variable with i =1, 2, 3… N variables

and t = 1, 2, 3,.,T Time

$B_{k,i}$ shows the k[th] coefficient of an explanatory variables or Proxy variables in cross sectional regression.

$X_{k,,t}$ explanatory variables or independents variables, at time t of k independents variables for i dependent variables

$\varepsilon_{i,t}$ Independent and identical distributed error term with zero mean and variance $\sigma^2\epsilon_{i,t}$.

Thus, we have the following set of equations: [16].

$$Y_{1,t} = B_{0,1} + B_{1,1}X_{1,t} + B_{2,1}X_{2,t} + \cdots + B_{k,1}X_{k,t} + \varepsilon_{1,t}$$

$$Y_{2,t} = B_{0,i} + B_{1,2}X_{1,t} + B_{2,2}X_{2,t} + \cdots + B_{k,2}X_{k,t} + \varepsilon_{2,t} \quad \dots(2)$$

$$Y_{N,t} = B_{0,N} + B_{1,N}X_{1,t} + B_{2,N}X_{2,t} + \cdots + B_{k,N}X_{k,t} + \varepsilon_{N,t}$$

The **second step** can be regarded as a cross-sectional regression model as follows

$$Y_{i,t} = \gamma_{0,t} + \gamma_{1,t}\hat{B}_{1,i,t} + \gamma_{2,t}\hat{B}_{2,i,t} + \cdots + \gamma_{K,t}\hat{B}_{K,i,t} + u_{i,t} \quad \dots(3)$$

Where $Y_{i,t}$ represents the dependent variable for unit i at time t , i =1, 2, 3… N variable (units) and t = 1, 2, 3,.,T Time.

$\gamma_{0,t}$ is the intercept of the cross-sectional regression at time t.

$\gamma_{1,t}, \gamma_{2,t}$, …, $\gamma_{k,t}$ are the coefficients representing the effect of independent variables on dependent variables (in period t) for the K factors.

$\hat{B}_{1,i,t}, \hat{B}_{2,i,t} \dots, \hat{B}_{k,i,t}$ are the estimated factor sensitivities (betas) for unit i, obtained from the first step time-series regression.

$u_{i,t}$: Is independent and identical distributed error term with zero mean and variance $\sigma^2 u_{i,t}$. [3].

This process involves running a cross-sectional regression for the individual dependent variable against the factor proxy or beta as independent variable, at each point in time in order to obtain time series of gamma for each factor, and the averages of these coefficients to obtain the parameter of FM regression and testing if these are statistically significant [4],[16].

Cross-sectional data, by definition, illustrates the extent of variation in the value of a variable from one item to another at the same point in time. This concept is visualized in Table (1), which depicts a general structure of cross-sectional data across various time periods.

From these cross-sectional regressions equ (3) implement for each cross sectional in table (1), we obtain T estimates for the gamma (γ) for each factor. Since t ranges from 1 to T, the number of cross-sectional regressions performed is T. This process yields time series of estimated γ coefficients, as summarized in Table (2).

Table (1) Cross-sectional Data Structure

Cross sectional at point time t	Dependent variable	Independent variable		
t=1	$Y_{1,t=1}$	$\hat{B}_{1,1,1}$	$\hat{B}_{2,1,1}$	$\hat{B}_{k,1,1}$
	$Y_{2,t=1}$	$\hat{B}_{1,2,1}$	$\hat{B}_{2,2,1}$	$\hat{B}_{k,2,1}$
	$Y_{3,t=1}$	$B_{1,3,1}$	$B_{2,3,1}$	$B_{k,3,1}$
	$Y_{N,t=1}$	$\hat{B}_{1,\boldsymbol{N},1}$	$\hat{B}_{2,\boldsymbol{N},1}$	$\hat{B}_{k,\boldsymbol{N},1}$
t=2	$Y_{1,t=2}$	$\hat{B}_{1,1,2}$	$\hat{B}_{2,1,2}$	$\hat{B}_{K,1,2}$
	$Y_{2,t=2}$	$\hat{B}_{1,2,2}$	$\hat{B}_{2,2,2}$	$\hat{B}_{K,2,2}$
	$Y_{3,t=2}$	$\hat{B}_{1,3,2}$	$\hat{B}_{2,3,2}$	$\hat{B}_{K,3,2}$
	$Y_{N,t=2}$	$\hat{B}_{1,\boldsymbol{N},2}$	$\hat{B}_{2,\boldsymbol{N},2}$	$\hat{B}_{K,\boldsymbol{N},2}$
t=T	$Y_{1,t=T}$	$\hat{B}_{1,1,T}$	$\hat{B}_{2,1,T}$	$\hat{B}_{K,1,T}$
	$Y_{2,t=T}$	$\hat{B}_{1,2,T}$	$\hat{B}_{2,2,T}$	$\hat{B}_{K,2,T}$
	$Y_{3,t=T}$	$\hat{B}_{1,3,T}$	$\hat{B}_{2,3,T}$	$\hat{B}_{K,3,T}$
	$Y_{N,t=T}$	$\hat{B}_{1,\boldsymbol{N},T}$	$\hat{B}_{2,\boldsymbol{N},T}$	$\hat{B}_{K,\boldsymbol{N},T}$

Table (2) Cross-sectional Regression Coefficients

	intercept	Regression coefficients		
Cross sectional regression coefficients	$\gamma_{0,t=1}$	$\gamma_{1,t=1}$	$\gamma_{2,t=1}$	$\gamma_{K,t=1}$
	$\gamma_{0,t=2}$	$\gamma_{1,t=2}$	$\gamma_{2,t=2}$	$\gamma_{K,t=2}$
	⋮	⋮	⋮	⋮
	$\gamma_{0,t=T}$	$\gamma_{1,t=T}$	$\gamma_{2,t=T}$	$\gamma_{K,t=T}$
Total of each column	$\sum_{t=1}^{T} \gamma_{0,t}$	$\sum_{t=1}^{T} \gamma_{1,t}$	$\sum_{t=1}^{T} \gamma_{2,t}$	$\sum_{t=1}^{T} \gamma_{K,t}$
Fama Macbeth regression estimates	$\bar{\gamma}_{0,t} = \frac{1}{T}\sum_{t=1}^{T} \gamma_{0,t}$	$\bar{\gamma}_{1,t} = \frac{1}{T}\sum_{t=1}^{T} \gamma_{1,t}$	$\bar{\gamma}_{2,t} = \frac{1}{T}\sum_{t=1}^{T} \gamma_{2,t}$	$\bar{\gamma}_{k,t} = \frac{1}{T}\sum_{t=1}^{T} \gamma_{K,t}$

Subsequently, a t-test of these time-series estimates gamma is computed to determine if the average gamma over time for each factor is significantly different from zero, when the statistic test is:

$$t_{statistic} = \frac{\bar{\gamma}_{k,t}}{\sigma_{\gamma_{k,t}}/\sqrt{T}} \qquad \dots (4)$$

3-literature Review

The Fama-MacBeth model has been widely used in literature. Below is a review of some notable previous studies that have used this model or related methods.

Fama, E. F., & MacBeth, J. D. (1973)[11] This study aimed to examine the empirical relationship between stock returns and risk, based on a two-parameters portfolio model and market equilibrium models. The study used a two-steps regression methodology: estimating risk measures (beta) for each security across time series (constant beta) (step 1), then conducting cross-sectional regressions within each time period to estimate risk premiums (step 2), using the ordinary least squares method. The main conclusion was that stock pricing reflects investors' pursuit of efficient portfolios. The results also showed that the "fair play" properties of the regressions are consistent with an efficient capital market, where prices fully reflect available information, and supported the positive relationship between systematic risk (beta) and expected returns.

Fama, E. F., & French, K. R. (1993)[10] They expanded the Capital Asset Pricing Model (CAPM) by incorporating two additional factors size and book-to-market ratio along with a market risk factor to better explain stock returns. This is a widely used asset

pricing model and portfolio management model. They estimated the model through time series analysis using ordinary least squares as an estimation method. They concluded that market risk alone was insufficient to explain all variations in stock returns. They found that adding these two factors significantly enhanced the explanatory power of the model. The **main difference** between this model and the Fama-MacBeath (1973) methodology is that the Fama-French model provides a specific theory that identifies the factors that determine stock returns, while the Fama-MacBeath methodology is a general two-step statistical procedure used to test whether any set of factors, including the Fama-French factors, are actually priced in the market and affect expected returns across a variety of assets. In other words, Fama-French provides the "what" (the factors), while Fama-MacBeth provides the "how" (these factors are analyzed and tested).

Shanken, J., & Zhou, G. (2007)[32]evaluated the performance of the Fama and MacBeth (1973) two-step procedure for estimating cross-sectional expected return models by simulating and comparing it with maximum likelihood (ML) and generalized method of moments (GMM) estimators. The Fama and MacBeth procedure was used as one of the primary methods for estimating these models. The study found that the generalized least squares (GLS) estimator is often more accurate than the ordinary least squares (OLS) estimator but suffers from greater bias. A "truncated" form of the ML estimator also performed well in terms of bias and accuracy but provided less reliable inferences than the OLS estimator.

Doan, M. P. (2011)[8] investigated whether systematic skewness are important in pricing factors for asset returns and investigated using both time series and cross-sectional analysis approaches. The Fama and French (1992) methodology is used to examine the time series returns while the Fama and Macbeth two-step method estimated by ordinary least squares with a five-year window size is used to examine asset returns in the cross-sectional analysis. Interestingly, the study found that when systematic skewness and skewness are added to the capital asset pricing model, they appear to be the dominant explanatory variables and render the market factor insignificant.

Ferreira, Gil-Bazo, and Orbe (2011)[12] estimated the first step by kernel weighted regression (with window size based on bandwidth) and the second step in Fama Macbeth estimate by Smooth Feasible Generalized least squares (SFGLS) and the bandwidth select by Generalized Cross Validation method GCV, and compare with OLS. The study demonstrates that nonparametric beta estimates outperform traditional estimates, reducing pricing errors and enhancing the accuracy of estimates. Monte Carlo simulations are also used to validate the effectiveness of the Fama and French three-factor conditional model, demonstrating that the nonparametric approach provides a better fit of the model to the data compared with OLS estimation method.

In **Ketsiri's (2012)[17]**study, the relationship between weather exposure, weather risk, and stock returns in the US market was investigated over the period 1980 to 2009, within the framework of arbitrage pricing theory. The study focused on three particularly weather-sensitive industries. To estimate the weather risk return, the researcher applied the two-steps methodology of Fama and MacBeth (1973), estimated using ordinary least squares (OLS). The results provided weak support for a statistically significant market price for weather risk in the unconditional framework, although the magnitude of the risk returns was relatively high. Most of the weather price estimates indicated negative values, which may warrant insuring stocks exposed to sudden temperature increases. The results showed good robustness, but the significance of the weather risk return was slightly affected by the model specifications.

Chee, W. Y. (2012)[6] examined whether abnormal volatility and extreme returns are priced into the Japanese stock market. Using the Fama and French three-factor model, Fama-Macbeth (1973) cross-sectional regression estimated by the ordinary least squares method, and pairwise classification of

stocks into portfolios based on variables of interest to test this effect. The results of the study identified four main findings about individual volatility and extreme returns in the Japanese stock market. First, the result showed a strong negative trend in individual and market volatility in the Japanese stock market between 1980 and 2007. Second, the research results confirm that volatility, whether equal-weighted, value-weighted, individual volatility, market volatility, or daily extreme returns, is unable to predict one-month excess market returns. Third, the result showed a negative relationship between individual volatility and expected stock return and an inverse relationship between daily extreme returns and expected stock return. Finally, the result showed a highly significant inverse relationship between individual volatility, maximum daily return and cross-sectional returns.

Dondeti, V. R., & McGowan Jr, C. B. (2013)[8] : used the Fama-MacBeth regression analysis methodology estimated by the ordinary least squares method to determine whether twenty indices for the twenty-year period from 1990 to 2009 provide a linear relationship between index returns and index betas. The empirical results are found to be inconsistent with the assumptions of the capital asset pricing model. For the time period and sample of indices, the empirical results based on gamma tests of the relationship between index returns and index betas do not support the capital asset pricing model.

Gempesaw, D. C. (2014)[14] used different samples of portfolios (Fama-French portfolios formed on the basis of size and book-to-market value, Fama-French industrial portfolios, and exchange-traded funds) as test assets to investigate whether the negative relationship between lagged individual volatility (IVOL) and future average returns is due to a missing risk factor using the Fama-MacBeth model estimated using the ordinary least squares method. Analytically, we show that if IVOL is a missing risk factor, the negative relationship between IVOL and returns persists at the portfolio level because systematic risk is not eliminated by diversification. However, when we take it into the data, we find no economically and statistically significant evidence of a relationship between lagged IVOL and subsequent average returns. Together, our results suggest that the IVOL puzzle is not due to a missing risk factor.

Nieto, B., Orbe, S., & Zarraga, A. (2014)[29]: compared the performance of least-square estimators including nonparametric weights GARCH-based estimators and Kalman filter estimators to estimate two steps of Fama Macbeth, test significant of CAMP model in Mexican stock market (2003-2009). Results show that Kalman filter estimators with random coefficients outperform the other methods.

Bai, J., Zhou G., (2015)[3]: proposed an estimate of Fama MacBeth two steps by adjusted OLS, GLS and compare with standard OLS, GLS. The demonstrate analytically and using simulations that the standard OLS and GLS estimators can contain large bias, but the adjusted OLS and GLS estimators can reduce the bias significantly.

Rask (2016)[31] aimed to develop the existing conditional asset pricing model of Dickson (2015), which identified book-to-market, market capitalization, total profitability, investment, short-term reversion, and momentum as the best explainers of stock returns. Rask (2016) used Fama-MacBeth regressions, based on the least squares method with a five-year window size, to assess whether the added risk premium could be better explained by each of the firm characteristic's variables. The study concluded that the Dickson (2015) model can better explain risk premiums when allowing for interaction terms and nonlinear functional forms. The study also showed that including the interaction between investment and market value improves the explanatory power of both investment and market value variables.

Green, J., J. Hand, and F. Zhang, (2016)[15] analyzed the predictability of stock returns using 94 financial characteristics as independent variables for the period from 1980 to 2014. They used Fama-MacBeth regression to select 12 influential independent variables

and estimated them using ordinary least squares (OLS) and value-weighted least squares (VWLS) regression on all stocks to estimate a five-year window size. The study responded to Cochrane's (2011) challenge to identify firm characteristics that provide independent information about average monthly US stock returns by simultaneously introducing 94 characteristics into Fama-MacBeth regressions while avoiding overweighting small firms and adjusting for data mining bias. The study found that while 12 characteristics were reliable independent determinants of returns in non-small-cap stocks over the period 1980 to 2014 as a whole, the predictability of returns declined sharply in 2003, with only two characteristics remaining as independent determinants since then. The study also noted that hedging returns resulting from exploiting characteristics-based predictability have been statistically insignificant outside of small-cap stocks since 2003.

Yahya, H. (2016)[34] the Fama-MacBeth (1973) methodology and the generalized method of moments with a five-year two-step estimation window were used to price the liquidity factor in both investable and non-investable indices. The HFR database indices for both types were examined, and the results showed that the exposure of these indices to the liquidity factor is largely determined by their characteristics and construction methods. This was explained by the influence of other factors within the Fung and Hsieh seven-factors model, which showed that when certain characteristics were controlled, the index's exposure to the liquidity factor could become statistically insignificant. The study also showed that investable indices are poor estimators of hedge fund performance in general, rejecting the null hypothesis of the zero-alpha test. The study emphasized the importance of the liquidity factor in the hedge fund industry due to the illiquid nature of its investments, finding that the liquidity factor is a priced factor in both investable and non-investable funds, with a significant liquidity premium even after controlling for autocorrelation.

Wang Xiaoxi (2017)[33] analyzed the predictability and anomaly of stock returns, using 40 financial and accounting characteristics as independent variables for the period 1980–2016. Macbeth regression was performed using the least squares estimation method with a five-year window size. To examine the persistence of anomalies over time and identify characteristics that provide independent information, they found a decrease in the number of significant characteristics in the period 2000–2016 compared to the period 1980–1999. They used the least squares estimation method with a five-year window size for feature selection. They identified a few significant characteristic

Forrester, A. S. (2017)[34] studied the role and nature of macroeconomic shocks and their relationship to the cross-sectional variance of asset returns, using the Fama-MacBeth procedure estimated using ordinary least squares with standard error correction and a five-year estimation window. The results indicated that shocks to relatively stable or less volatile macroeconomic indicators are positively and significantly priced. The study concluded that the information contained in common risk factor shocks is significantly priced across the cross-section of asset returns and differs from the information contained in the Fama-French-Carhart factors.

Liu, Y. (2017)[21]analyzed the effects of capital constraints on monetary policy transmission, using a Fama-MacBeth regression estimated using ordinary least squares with a five-years estimation window, to examine the effect of bank capital constraints on the transmission mechanism. The model showed that transmission is stronger (loan supply is more sensitive to changes in monetary policy) if banks are well-capitalized. A two-step Fama-MacBeth empirical test confirmed that changes in monetary policy are more significant for lending to banks with higher capital ratios; and capital constraints intensify during periods of monetary policy tightening. Both effects are largely attributable to smaller banks.

Abeland, M., and Petersen, M. J. (2018)[2]they investigated the compensation of

US investors for bearing exchange rate risk in international exchange-traded funds (ETFs), using two currency risk factors: "dollar" and "carry." The study covered the period from January 1997 to June 2015. To estimate risk exposure and conditional prices, they relied on the Fama and MacBeth two-steps methodology, applying the ordinary least squares (OLS) method. In the first step, ETF sensitivities to currency risk factors were estimated using time-series regressions across rolling windows. In the second step, monthly cross-sectional regressions were conducted on the excess returns on these estimated sensitivities. The researchers concluded that US investors are indeed compensated for bearing currency risk, particularly interest rate spread trading risk. The study demonstrated that models combining equity and currency risk factors outperformed models that relied solely on the equity factor at explaining ETF excess returns.

Kim Bae Wong, 2018[19] tested the validity of various asset pricing models in two different financial markets, the US and the Korean markets, using stock market data. The main objective of this thesis is to determine whether asset pricing models are empirically sound across different markets and time periods, as well as to compare their explanatory power. The Fama-MacBeth method was used to test the validity of the capital asset pricing model, and Jensen's alpha was used to validate the factor models. For reference, the Fama-MacBeth regression was estimated using the ordinary least squares method. They concluded that the factor models developed by Fama-French (1993, 2015) are pioneering models that incorporate empirical evidence of asset size and value into the asset pricing model.

Lønø, B. E., & Svendsen, C. E. (2019)[22]: used the ordinary least squares (OLS) method to estimate the Fama-MacBeth steps to evaluate different asset pricing models for the Norwegian stock market. The study applied four models to the Norwegian stock market: the capital asset pricing model, the Fama-French three-factor model, the Fama-French five-factor model, and the Carhart four-factor model. The purpose of the comparison was to find a better model that could be applied to the Norwegian stock market. The study found that the Fama-French three-factor model is a relatively stable and applicable model.

Yan, J. (2019)[35] The information content of publicly available short-term flows was investigated. Although daily short-term flow data are published in real time over the period 2010–2018, using Fama-MacBeth regressions estimated with ordinary least squares, a simple sort based on the short volume ratio was found to be a significant predictor of negative future stock returns**.**

Chen, Y. M. (2019)[6] Fama-MacBeth estimated the information content of insider trading in financial markets using the least squares method. By analyzing the private information transmitted during a long period of insider silence and its relationship with investor behavior and abnormally low returns in distressed stocks, he explored the causal relationship between the well-documented financial distress anomaly from the perspective of investor behavior. And by analyzing the mechanism of insider trading as a signaling tool in an attempt to solve the puzzle of the benefits and costs of insider trading activity.

Markowski, L. (2020)[24] The ordinary least squares method was used to estimate the Fama-MacBeth. The aim was to validate the capital asset pricing model (CAPM) in the Polish capital market based on the traditional approach and the downside risk approach. The CAPM was found to be insignificant. The researcher compared the unconditional and conditional return relationships, estimated using realized returns in cross-sectional regressions, using the beta coefficient. Except for the beta coefficient, the CAPM model was tested using the common skewness coefficient as the higher-order participation coefficient and the negative beta as the downside risk measure. The unconditional regressions provided evidence of a risk premium associated with the common skewness coefficient and the downside beta, and confirmed the validity of the downside CAPM model**.**

Liu, R. (2021)[21]: estimate Fama and MacBeth (1973) cross-sectional regressions, using the ordinary least squares method to

formally assess the effect of skewness robust and skewness robust across all cross-sections while controlling for volatility. That is, to analyze the relationship between stock returns as a dependent variable and skewness and volatility as independent variables.

Nguyen, V. D. (2021)[28] investigate the effect of product market competition on asset growth anomalies, a general asset pricing anomaly. Asset growth anomalies are the negative relationship between asset growth and future stock returns. Whether the phenomenon can be explained by the risk hypothesis or the mispricing hypothesis is discussed. Product market competition is used as a control variable using three methods: time series, a Fama-MacBeth least squares model estimated with a five-years window size, and dual portfolio analysis. The empirical results confirm the role of market competition in the relationship between asset growth and future stock returns**.**

Kroon, E., & Karlsson, T. (2021)[20]: The purpose of this study is to examine whether the retail trade imbalance anomaly can be replicated on small, listed Swedish firms. We use Fama-MacBeth cross-sectional regressions estimated using the ordinary least squares method with a five-year window size. The concludes that the retail trade imbalance anomaly cannot be replicated when applied to the selected environment.

Khazanov, A. (2022)[18]: studied sovereign default risk and currency returns, i.e., foreign currency returns in a cross-section of countries are positively related to sovereign default risk. To formalize the link between default risk and currency returns, he quantified a "with-default" model for a set of developing countries. He then used the effects of this model to construct an econometric model of the currency returns cross-section, which he estimated using the Fama-MacBeth least-squares model with a five-year window size. He found that credit default swap spreads, which act as a proxy for default risk, account for about 25% of the cross-country variation in average currency returns. He also estimated that market participants expect a 50% depreciation of the national currency upon default.

Baillie, Calonaci, and Kapetanios (2022)[4]: aimed to test the significance of the Fama-French three-factors model's factors on S&P 500 financial return datasets. The researchers employed the sequential Fama and MacBeth (1973) methodology to estimate multifactor dynamic asset pricing models. Their approach notably featured the use of weighted kernel regressions and various flexible bandwidth selection methods. This allowed for more accurate and time-varying estimates of risk premiums and factor loadings. The study's findings indicated that utilizing a cross-validation procedure yielded consistent estimates and led to a statistically significant improvement in the predicted loss function measures, irrespective of the specific factor included in the analysis.

Persson, O., & Lindblom, S. (2024) [30]This study examines the relationship between the three measures Price-to-earnings ratio dividend yield, and debt-to-equity, and the European stock market between January 2010 and December 2022. This is done using a two-step regression model proposed by Fama and Macbeth estimated using the ordinary least squares method with a window size of five years. these results show that there is an anomaly in the European stock market and that there is a company-specific risk associated with these measures. This suggests that when looking at individual companies, investors are willing to pay a premium for the measures studied, and it is therefore important to take them into account when examining individual companies.

Aghabeigi, M., & Ondes, T. (2024)[1]: aimed to evaluate the relationship between corporate governance factors and stock returns based on the capital asset pricing model and the three-factors model of Fama and French. The panel data method was used along with the two-step's regression of Fama and Macbeth estimated by the ordinary least squares method with a window size of five years. The results represented a significant and positive relationship between the general index of

corporate governance and the stock returns of companies.

Mousa, M. J., & Hmood, M. Y. (2025)[25]. This study aimed to use the Single-Index Model to develop and modify the Fama-MacBeth model. This modification was achieved using the Penalized Smoothing Spline Regression technique, abbreviated as SIMPLS.Two methods of Generalized Cross-Validation were used to determine the optimal value of the smoothing parameter: the Generalized Cross-Validation Grid (GGCV) and the Generalized Cross-Validation Fast (FGCV).Due to the two-step nature of the Fama-MacBeth model, the estimation process produced four different estimators by combining the cross-validation techniques :SIMPLS(FGCV)-SIMPLS(FGCV),SIMPLS(FGCV) - SIMPLS(GGCV),SIMPLS(GGCV) - SIMPLS(FGCV),SIMPLS(GGCV) - SIMPLS(GGCV)

The study estimated the three-factor Fama-French model (market risk premium, size factor, and value factor) and their impact on the excess returns of stocks and portfolio returns in the Iraq Stock Exchange, using the modified Fama-MacBeth model.

The results showed that the SIMPLS(FGCV)-GGCV estimator had the best performance. The results also revealed the statistical significance of the three Fama-French model factors, which enhanced the model's explanatory power regarding the performance of the Iraq Stock Exchange.

Mousa, M. J., & Hmood, M. Y. (2025)[27] This paper addresses the estimation of cross-sectional regressions in cases where model parameters change over time and the independent variables serve as proxies. To achieve this, the Fama-MacBeth (FM) technique was applied using the semiparametric Single-Index Model (SIM) and nonparametric Kernel-Weighted Regression (KWR). Consistent estimates for the model parameters and proxy variables were obtained by employing the Generalized Cross-Validation (GCV) method, a time-varying technique for selecting the smoothing parameter.

During the estimation process, four distinct methods were developed for estimating the Fama-MacBeth model by combining the mentioned techniques: KWR-SIM, SIM-KWR, SIM-SIM, and KWR-KWR. These methods were applied to data from the Iraq Stock Exchange using the three-factor Fama-French model. The results showed that the KWR-SIM approach achieved the best performance in terms of model fit to the data.

Mousa, M. J., & Hmood, M. Y. (2025)[26] This study presents a novel approach to improving Fama-MacBeth (FM) model estimations by utilizing the Dragonfly Algorithm (DA) and the Fruit Fly Algorithm (FA), comparing their performance for the first time in this context.

To specifically enhance the performance of the Dragonfly Algorithm, the study explored three parameter tuning methods: Manual Parameter Tuning (MPT), Adaptive Tuning based on Methodology (ATY), and a new proposed Adaptive Parameter Tuning based on Performance (APT) method. The study also compared the performance of these approaches with the traditional Kernel Weight Regression (KWR) method. Additionally, the study proposed using both the Dragonfly and Fruit Fly algorithms to improve the parameters of the KWR method itself.

These methods were applied empirically to data from the Iraq Stock Exchange using the three-factor Fama-French model. The results showed a significant superiority of the Dragonfly Algorithm (when using the MPT and APT methods) in improving the accuracy of the Fama-MacBeth model's estimations and enhancing the performance of the KWR method.

4. Conclusions

Artificial intelligence algorithms, such as the Dragonfly algorithm, have also been employed to improve the accuracy of estimates. This methodological development has made the model more capable of handling the complexities of modern financial data.

A comprehensive literature review shows that the Fama-MacBeth model has evolved significantly from a simple tool for testing the capital asset pricing model (CAPM) to a flexible and powerful statistical framework used today in a wide range of financial research. Initially, the model relied primarily on ordinary least squares (OLS). However, with the emergence of statistical challenges such as estimation bias, recent studies have begun to shift toward more sophisticated estimation methods such as generalized least squares (GLS), Kalman filters, modified FM models using a single-index model, and estimation using partitioned least squares (PLS) regression estimators. Artificial intelligence algorithms, such as the Dragonfly algorithm and the Fruitfly algorithm, have also been employed to improve the accuracy of estimates. This methodological development has made the model more capable of handling complex data.

The model's use is no longer limited to testing traditional risk factors such as market beta. It has expanded to include the analysis of new and complex factors, such as liquidity risks, macroeconomic shocks, firm characteristics, and even investor behavioral factors. This expansion demonstrates the model's flexibility and adaptability to developments in financial research. Therefore, the research confirms that the Fama-MacBeth model is not merely a historical concept, but a revitalized research tool. Its methodological developments and growing applications have made it a solid foundation for analysis in the field of asset finance.

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